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ON THE PROBLEM OF THE INTERNAL CONSTITUTION OF THE EARTH

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ON THE PROBLEM OF THE INTERNAL CONSTITUTION OF THE EARTH

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Abstract: *In this work a mathematical model on the problem of the internal constitution of the Earth is established. Essentially the fundamental hypothesis that serve as a basis for the theory developed in our work are the following: Sphericity of the Earth, refraction and reflection of P and S waves, internal stratification of the Earth into five spherical regions, and finally a velocity law for the transmission of the P and S waves in each stratum. A comparison between epicentral distances and travel-times obtained for this model shows very good agreement with well-known tables derived from direct observations.*

1. INTRODUCTION.

Astronomical and geodesic studies have given numerous and significant dynamic and geometric data about our planet, among which we can denote its shape, its dimensions, its movements, its total mass, its mean density, the acceleration of gravity, etc.

(*) Although the basic ideas and the majority of the developments of this work, together with the calculations that accompany it, have been evolved by Y. Lana-Renault, we should like to point out that Prof. R. Cid Palacios has made an estimable contribution with his suggestions in systematizing, simplifying and improving its content.

The terrestrial constants that have been obtained as a consequence of such studies can be summarized in the following way:

Equatorial radius R_e = 6.378.137 m.

Flattening f = 1 / 298,257

Mean radius R_0 = 6.371.028 m.

Total mass M = 5.977 x 10^{27} gr.

Mean density ρ = 5,517 gr./cm^3.

Gravitational constant G = 6,672 x 10^{-11} $m^3kg^{-1}s^{-2}$.

Surface gravity g_0 = 979,981 gals.

Many other results are owing to physicists, chemists, geologists and geophysicists, whose enumeration would be too lengthy to be included in this brief outline, although the majority of such data refer to the physical state and composition of the Earth's surface.

The physical state and the composition of the Earth's interior is evidently much less well known, Geophysics perhaps being the science which has provided the most reliable data about its interior .

In this search the installation of a world-wide standardised seismographic network has occupied a preponderant position and it has permitted the establishment of a certain number of conclusions.

The foundation of these conclusions is principally based on the study of the propagation of elastic waves through the interior of the Earth and fundamentally to two types of waves: P waves and S waves. The first are due to the transmission of compressions and expansions, the second to shearing movements. Agreeing with this, and based on a certain number of properties concerned with the propagation, refraction, reflection and diffraction of waves, as well as with the travel-time employed by the said waves in their transmission across the interior of the Earth, more or less concordant results have been obtained.

Thus it is accepted that the velocities v_p and v_s at which the P and S waves are transmitted, are determined by the relationships

$$v_p = \sqrt{(3\kappa + 4\mu) / 3\rho} \qquad\qquad v_s = \sqrt{\mu / \rho}$$

where κ represents the incompressibility modulus, μ the rigidity modulus and ρ the density at each point. According to this, it is evident that $v_p > v_s$ and that the S waves cannot be propagated across the fluid medium.

Likewise, tables have been elaborated (Jeffreys and Bullen, 1958; Heerin and others, 1968; Randall, 1971) which provide the travel-times T employed by the rays of the P and S waves in their propagation, from an initial point P_o (where they have been produced), to a point Q_o at which they are registered by observation at the Earth's surface, as functions of the angle or epicentral distance $\Delta = P_oOQ_o$, which determine the points P_o and Q_o in relation to the centre O of the Earth.

Owing to these studies the division of the interior structure of the Earth into seven regions (A, B, C, D, E, F, G) has been achieved, which M.H.P. Bott (1971), in an adaptation from the layering given by K.E. Bullen (1963), outlines in the following way:

	Region	Depth range (Km.)	
CRUST	A	0-33 (variable thickness)	
Mohorovicic discontinuity			
	B	33-400	upper mantle
MANTLE	C	400-1000	transition zone
	D	1000-2900	lower mantle

Gutenberg discontinuity			
	E	2900-4980	outer core
CORE	F	4980-5120	transition zone
	G	5120-6370	inner core

According to this internal constitution of the Earth, a standard nomenclature has been elaborated, which permits the definition of the different phases of an earthquake and the principal kinds of waves transmitted that we consider in this essay.

Types of waves	Definition of their trajectories
P, S,	P or S waves across the mantle.
PP, SS, PPP, SSS, etc.	P or S waves transmitted across the mantle and reflected once or more at the Earth's surface.
PS, SP, PPS, SPP, etc.	Waves across the mantle, reflected at the Earth's surface once or more and changing their type.
PcP, ScS, PcS, etc.	Waves across the mantle reflected at the surface of the outer core, changing or not changing their type.
PKP	Waves transmitted through the mantle, outer core and mantle.
PKiKP	Waves transmitted through the mantle and outer core and reflected at the surface of the inner core toward the Earth's surface.

PKIKP	**Waves transmitted through the mantle, outer core and inner core, returning toward the Earth's surface.**

Other considerations and observations have permitted the determination of certain parameters for the Earth's interior (density, pressure, temperature, potential, gravity, etc.) based on the distance r to the centre of the Earth, contrasting with the fact that, in principle, the graphs of such functions $\rho(r)$, p(r), etc., do not seem to satisfy any mathematical law.

It is evident that in its secular process of formation, our planet must have suffered alterations that render difficult the establishment of a unified theory about its constitution; but it is no less true that, disregarding local irregularities, one can conceive of a mathematical model whose formulation is in approximate accordance with the reality of such an internal constitution and which satisfies the observational data appreciably well.

Although the establishment of such a formulation is no more than an approximate reality, it has, on the other hand, a great deal of didactic interest, as well as that of systematization, these being the principal motives that have led us to write this work.

With that in mind, we shall begin by establishing some fundamental hypothesis that will serve as a basis for the theory developed in this work, not with the intention of making such hypothesis axiomatic in character, but rather with the aim of facilitating a contrast of its consequences with experimental reality in such a way that the results of our formulation agree with the measurements, brought together in several tables, and carried out by numerous observers.

The fundamental hypothesis to which we refer are, in principle, the following:

1. It is assumed that the Earth is a sphere whose mean radius is R_0 = 6.371.028 m., with spherical stratification, in such *a manner that the velocity v of the waves, the density ρ, the pressure p, etc., at a point P depend exclusively on the radius r = OP (distance from the point P to the centre O of the Earth).*

It is undoubtable that the extension of this hypothesis to include the case of an ellipsoidal Earth, would lightly modify some results, although we estimate that its incidence would not be fundamental.

2. It is assumed that the Earth's interior is stratified into five spherical stratum E_0, E_1, E_2, E_3, E_c, which we shall denominate respectively, upper mantle, lower mantle, outer core, transition zone and inner core, whose properties and limits will have to be determined.

This division corresponds approximately to the structure denoted by M.H.P. Bott (1971) and adjusted to the observation of the paths of the P and S waves, which we have described before. The radius that limit these stratum will be denoted in the following way:

E_0:	Upper mantle	(R_0, R_0')
E_1:	Lower mantle	(R_1, R_1')
E_2:	Outer core	(R_2, R_2')
E_3:	Transition Zone	(R_3, R_3')
E_c:	Inner core	$(R_c, 0)$

Nevertheless, as it is assumed that the lower limit of one stratum coincides with the upper limit of the underlying stratum, the following equalities are verified:

$$R_0' = R_1 , \quad R_1' = R_2 , \quad R_2' = R_3 , \quad R_3' = R_c$$

Analogously, to follow a coherent notation, in a given stratum E_i, limited by the radius R_i and R_i', the angle that an incident ray forms with the radius R_i will be denoted by I_i, while the angle that it forms at its arrival with the radius R_i' will be I_i'. In this way, the angles that an incident ray forms across the whole of the interior of the Earth will be (I_0, I_0'), (I_1, I_1'), (I_2, I_2'), (I_3, I_3') and I_c.

Likewise, the conditions that determine the internal structure of the Earth, deduced from the existing tables, have led us to the conclusion that between the upper and lower mantle and between the outer core and the transition zone, discontinuities do not exist, for which reason we will admit in general the equalities $I_0' = I_1$, $I_2' = I_3$.

3. The trajectories of the seismic waves across the different stratum satisfy the laws of refraction, while at the upper and lower spherical surface, which limit a stratum, reflections of these waves can present themselves.

To clarify this, let us consider a stratum determined by its radius R and R' so that the radius of one of its points will verify the conditions $R \geq r \geq R'$.

The law of refraction for each wave that passes through this layer can be defined by means of the formula

$$p = \frac{r \sin I}{v}$$

where I is the angle that forms the tangent to the trajectory at a point P, with the radius $r = OP$, $v = v(P)$ the velocity and p a parameter. Its demonstration is analogous to that which defines astronomical refraction.

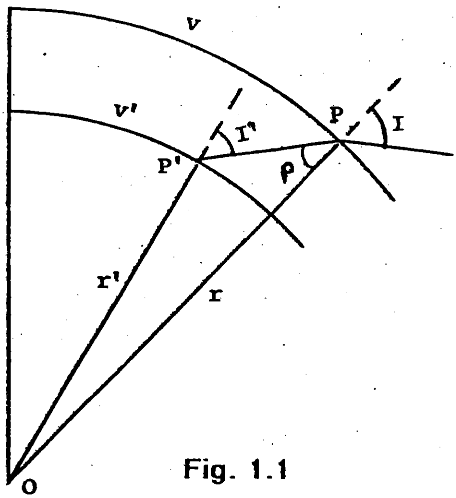


Fig. 1.1

In effect, let us consider a ray that falls upon a point P of its trajectory with an angle I, and with an angle I' upon a close point P'; being $r = OP$, $r' = OP'$, $v = v(P)$, $v' = v(P')$, the radii and the velocities corresponding with the said points.

If ρ is the angle of refraction at P, in the triangle OPP', one has (Fig. 1.1)

$$r \sin \rho = r' \sin I'$$

and according to the law of refraction, the following relationship is verified

$$v' \sin I = v \sin \rho$$

therefore, multiplying both equalities and eliminating the common term $\sin \rho$, one obtains the formula (1.1)

$$p = \frac{r \sin I}{v} = \frac{r' \sin I'}{v'}$$

Immediate consequences of this formula are the following:

a) At all points of a trajectory the parameter p is constant. That is to say, the different trajectories that leave from a point P_o, with initial radius r_o and velocity v_o, have a constant parameter p_o that depends exclusively on the angle of departure I_o.

b) The constancy of the parameter p along the whole of a trajectory P_oQ_o which, leaving from a point P_o of a surface of a radius r_o, arrives at another Q_o of the same surface, proves:

1.- That the angle I_o of departure and the angle of arrival are the same.

Strictly speaking, if we consider the angles that determine the directions P_oO and Q_oO with the tangents to the trajectory at P_o and Q_o, in the direction of the movement, these angles are supplementary.

2.- That if P_m is the deepest point of its trajectory, the curves P_oP_m and P_mQ_o are symmetrical.

3.- That the incident trajectory (and therefore the emergent one) is composed of a continuous curvilinear segment through a stratum, or of several segments through different stratum.

To this effect, the stratum (E_0,E_1) and (E_2,E_3), can be considered to be a single stratum, by virtue of the conditions $I_0' = I_1$, and $I_2' = I_3$, established before.

As is well known, given a trajectory P_oQ_o, the angle $\Delta = P_oOQ_o$ which the points P_o and Q_o determine with the center O of the Earth, receives the name of epicentral distance.

This parameter is usually accompanied by the travel-time T that the wave employs in describing the trajectory P_oQ_o.

In particular, the epicentral distance and the travel-time which determine an incident trajectory will be denoted by the symbols $\Delta^{(m)}$ and $T^{(m)}$, so $\Delta = 2\Delta^{(m)}$ and $T = 2T^{(m)}$.

c) Let us consider two trajectories P_oQ_o and P_oQ_o', which leave at a velocity v_o from the same point P_o, belonging to the surface of a radius r_o, and arrive at two close points Q_o, Q_o', of the same surface (Fig. 1.2). If $\Delta = P_oOQ_o$ and $\Delta' = \Delta + \delta\Delta = P_oOQ_o'$ are the epicentral distances that these trajectories determine, and T, $T' = T + \delta T'$ the corresponding travel-times we have

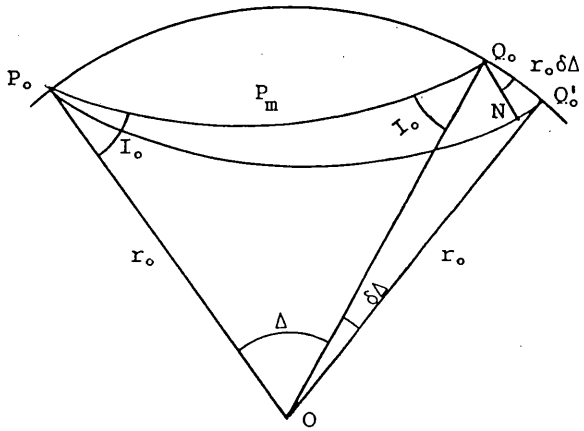


Fig. 1.2

$$p_o = \frac{r_o}{v_o} \sin I_o = \frac{\delta T}{\delta\Delta} \tag{1.2}$$

In fact, if for Q_o we trace a perpendicular Q_oN to the trajectory P_oQ_o, obviously, we have $NQ_o' = \delta s$ and $Q_oQ_o' = r_o\,\delta\Delta$ and therefore

$$\sin I_o = \frac{NQ_o'}{Q_oQ_o'} = \frac{\delta s}{r_o\,\delta\Delta} = \frac{v_o\,\delta T}{r_o\,\delta\Delta}$$

which proves (1.2).

The formula (1.2) established a relationship between the increments $\delta\Delta$ and δT, for different initial angles, independent of the conditions of transmission that the ray meets in its trajectory across the different stratum of the interior of the Earth.

d) If in a stratum determined by the radius (R,R'), a wave is reflected at the surface of radius R', it follows that

$$\frac{R' \sin I'}{v'} = \frac{R' \sin I''}{v''}$$

v" being the velocity of the reflected wave and I" the angle that the wave forms with the radius R'.

In this way, if a wave P (or S) is reflected like a wave P (or S), v' will be equal to v", and therefore $I' = I''$.

On the other hand, if a wave P is reflected like S, or the reverse, then $v' \neq v''$, and therefore $I' \neq I''$.

The properties established in the paragraphs *a), b), c), d),* define the general properties of wave transmission through the interior of the Earth.

The real existence of diffraction phenomenon in certain trajectories introduce small variations, difficult to treat, that will not be considered in this work.

2. DIFFERENTIAL FORMULAS THAT DEFINE THE FUNDAMENTAL PARAMETERS Δ AND T .

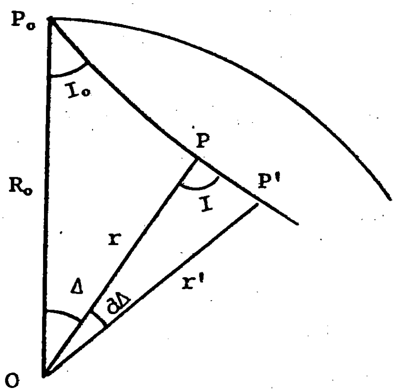


Fig. 2.1

If we assume a trajectory P_oPP' referred to the polar coordinates (r,Δ), in agreement with Fig. 2.1, an element of arc PP' = ds , allows us to write

$$\sin I = \frac{r\, d\Delta}{ds}$$

Therefore, if we put $\eta = r/v$ the following result is given

$$p = \eta \sin I = \eta\, r\, d\Delta / ds \qquad (2.1)$$

and furthermore, we have

$$ds = \sqrt{dr^2 + r^2\, d\Delta^2} = r \sqrt{(dr/r)^2 + d\Delta^2} \qquad (2.2)$$

Therefore, if we eliminate ds between (2.1) and (2.2), we obtain

$$d\Delta = \pm \frac{p}{\sqrt{\eta^2 - p^2}} \frac{dr}{r} \qquad (2.3)$$

Analogously, eliminating $d\Delta$ between (2.1) , (2.3) and the equality $ds = v\, dT$, we obtain

$$dT = \pm \frac{\eta^2}{\sqrt{\eta^2 - p^2}} \frac{dr}{r} \qquad (2.4)$$

In the formulas (2.3) and (2.4) , the sign (-) will be taken for the incident ray and the sign (+) for the emergent ray.

As a consequence of these differential formulas, the following equalities are easily obtained

$$dT - p\, d\Delta = \pm \sqrt{\eta^2 - p^2} \frac{dr}{r} \qquad (2.5)$$

$$\sin^2 I\, dT - p\, d\Delta = 0 \qquad (2.6)$$

It should be observed that the formula (2.6) is not equivalent to (1.2) , because in (2.6) the differentials dT and $d\Delta$ are related over a single trajectory, while (1.2) relates the increments δT and $\delta\Delta$ between two close trajectories that leave from the same point.

3. ADOPTION OF A VELOCITY LAW IN THE TRANSMISSION OF SEISMIC WAVES.

To establish a finite relation between Δ and T, it is necessary to integrate the equations (2.3) and (2.4) , in which the function $\eta = r/v$ appears. Ignorance of the velocity v(r) in function of the radius r, implies that the resolution of the problem set leads to integral equations, whose treatment has been studied by G. Herglotz (1907), H. Bateman (1910) and others, as K.E. Bullen (1963) has already pointed out. In general, its resolution has only been possible in a numerical form.

However, the form of the equations (2.3) and (2.4), suggests a possible and easy integration if we introduce a velocity law of the type $v = v(r, \log r)$.

Among the possible laws that can be adopted, we have chosen the one that seems the simplest to us, although its adoption depends on the results to which it leads. For that reason, in the present article we shall admit the following proposition, that completes the hypothesis established before:

4. For each stratum, a velocity law for the transmission of P and S waves is considered, defined by the formula

$$v(r) = r\,(B - A \log r) \tag{3.1}$$

where B and A are constants to be determined in each case.

It is supposed, in the same way, that there exists a small sphere of radius R_c, in such a way that, in its interior the velocity v_c is constant.

The introduction of this small sphere becomes necessary to avoid the singularity that the formula (3.1) presents for $r = 0$.

The adoption of this law of velocity distribution in the interior of the Earth will lead us to the establishment of numerous conclusions that we shall be specifying throughout the text and that will be contrasted with the observations made on the P and S waves.

With the exception of the inner core, the values of the constants B and A will be defined for each stratum in such a way that in the stratum E_i the said constants will be denoted by B_i and A_i.

On the other hand, we have found that our exposition is simplified if for each point in the interior of the Earth we define the function

$$k = v/r = 1/\eta = B - A \log r \tag{3.2}$$

In consequence, for the different stratum, we shall have

E_0: $k_0 = B_0 - A_0 \log R_0$ $k'_0 = B_0 - A_0 \log R'_0$

E_1: $k_1 = B_1 - A_1 \log R_1$ $k'_1 = B_1 - A_1 \log R'_1$

E_2: $k_2 = B_2 - A_2 \log R_2$ $k'_2 = B_2 - A_2 \log R'_2$

E_3: $k_3 = B_3 - A_3 \log R_3$ $k'_3 = B_3 - A_3 \log R'_3$

E_c: $k_c = v_c / R_c$

and the general form

E_i: $k_i = B_i - A_i \log R_i$

$k'_i = B_i - A_i \log R'_i = B_i - A_i \log R_{i+1}$

Evidently, the equality (3.1) can also be written in the form

$$v = r\,(B - A \log r) = A\, r\, (\beta - \log r)$$

putting $\beta = B / A$.

According to this, the equality (3.2) is equivalent to the following

$$k = A\,(\beta - \log r) \tag{3.2'}$$

The equivalent formulas (3.2) and (3.2') can be used without distinction throughout our exposition.

4. TRAJECTORY OF A SEISMIC RAY ACROSS ANY STRATUM.

Let us consider any particular generic stratum E_o, other than E_c, defined by the radii R_o and R'_o, and the constants A_o, B_o, so that the radius r(P) of one of its points will verify the conditions $R_o \geq r(P) \geq R'_o$. The subscript ($_o$) can be replaced by any one subscript 0, 1, 2, or 3.

The computation of trajectories across this stratum is based, essentially, on the formulas

$$k = B_o - A_o \log r \tag{4.1}$$

$$p_o = \frac{\sin I_o}{k_o} = \frac{\sin I}{k} \tag{4.2}$$

being $k_o = B_o - A_o \log R_o$, and I_o, I, the angles which the direction of the incident ray form with the respective radii R_o and r.

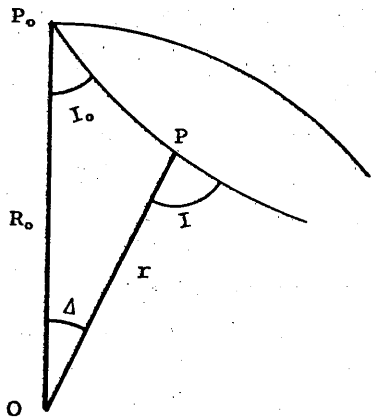


Fig. 4.1

The epicentral distance Δ and the travel-time T, between the points P_o and P of the trajectory, can be obtained by t integration of the formulas (2.3), (2.4), through a change of variable of the type $z = \log r$, that leads us to second degree irrational integrals; but their calculation is more easily obtained in the following way (Fig, 4.1):

In fact, let us consider the differential formulas

$$d\Delta = -\frac{dr}{r}\tan I \tag{4.3}$$

$$A_o \frac{dr}{r} = -\frac{\cos I \, dI}{p_o} \tag{4.4}$$

The first deduced from the trajectory on polar coordinates (r, Δ) and the second by differentiation of the equality

$$k = B_o - A_o \log r = \frac{\sin I}{p_o}$$

From the formulas (4.3) and (4.4), it is deduced that

$$d\Delta = \frac{\sin I \, dI}{A_o \, p_o} \tag{4.5}$$

and integrating between the points P_o and P, we have

$$\Delta(P_o,P) = \frac{\cos I_o - \cos I}{A_o \, p_o} \tag{4.6}$$

This equality can also be written in the forms:

$$\Delta(P_o,P) = \frac{1}{A_o}\left[\frac{k_o}{\tan I_o} - \frac{k}{\tan I}\right] \tag{4.6'}$$

$$= \frac{1}{A_o}\left[\frac{k_o}{\tan (I_o/2)} - \frac{k}{\tan (I/2)}\right] \tag{4.6''}$$

To calculate the travel-time employed in describing the trajectory between P_o and P, it is sufficient to remember the formulas (2.6) and (4.5), from which the following differential formula is easily obtained

$$dT = \frac{dI}{A_o \sin I} \tag{4.7}$$

Finally, integrating between P_o and P, one obtains

$$T(P_o,P) = \frac{1}{A_o} \log \left[\frac{\tan (I/2)}{\tan (I_o/2)}\right] = \tag{4.8}$$

$$= \frac{1}{A_o} \log \left[\frac{k\,(1 + \cos I_o)}{k_o\,(1 + \cos I)}\right] \tag{4.8'}$$

The anterior formulas are fundamental for the calculation of trajectories across any stratum E_0 , E_1 , E_2 and E_3 , once that corresponding values to the constants A_o and B_o have been assigned to it.

5. CONSEQUENCES OF THE ANTERIOR FORMULATION.

Let us now see some consequences of a general character that are obtained by applying the formulas of the previous paragraph to the trajectories that always pass through the same stratum.

a) Radius of any single point along the trajectory.

Integrating the formula (4.4) between P_o and P, we have

$$r(P) = R_o \exp\left(\frac{\sin I_o - \sin I}{A_o\, p_o}\right) = \tag{5.1}$$

$$= R_o \exp\left(\frac{k_o - k}{A_o}\right) \tag{5.1'}$$

that defines the corresponding radius to any point P. In particular, if P_m is the middle point of the total trajectory, for which one has $I_m = \pi / 2$, it turns out that

$$r(P_m) = R_o \exp\left(\frac{k_o}{A_o}\left(1 - \frac{1}{\sin I_o}\right)\right) = \tag{5.2}$$

$$= R_o \exp\left(\frac{1}{A_o}\left(k_o - \frac{1}{p_o}\right)\right) \tag{5.2'}$$

b) Epicentral distance $\Delta^{(m)}$ and travel-time $T^{(m)}$ corresponding to the trajectory (P_o, P_m).

Applying the formulas (4.6) and (4.8) to this case, and taking into account the relationships $\Delta = 2\Delta^{(m)}$, $T = 2T^{(m)}$, we can write the equalities

$$\Delta^{(m)} = \frac{\cos I_o}{A_o\, p_o} \tag{5.3}$$

$$T^{(m)} = -\frac{\log \tan (I_o/2)}{A_o} \tag{5.4}$$

c) Relationship between the parameters $\Delta^{(m)}$ and $T^{(m)}$.

A simple relationship between the variables $\Delta^{(m)}$ and $T^{(m)}$ can be obtained by means of the functions

$$\delta^{(m)} = A_o\, \Delta^{(m)} = \frac{k_o}{\tan I_o} \tag{5.5}$$

$$\tau^{(m)} = A_o\, T^{(m)} = -\log \tan (I_o/2) \tag{5.5'}$$

In fact, one has

$$\tan I_o = \frac{2 \tan (I_o/2)}{1 - \tan^2 (I_o/2)} =$$

$$= \frac{2\, e^{-\tau^{(m)}}}{1 - e^{-2\tau^{(m)}}} = \frac{2}{e^{\tau^{(m)}} - e^{-\tau^{(m)}}}$$

from where

$$\delta^{(m)} = k_o \operatorname{Sinh} \tau^{(m)} \tag{5.6}$$

d) <u>Possibility of the existence of a point P^* of a maximum velocity v^*.</u>

By derivation of the formula (3.1), with respect to r, we obtain

$$dv / dr = B_o - A_o \log r - A_o = k - A_o \tag{5.7}$$

Therefore, for the existence of a relative maximum of the velocity, the conditions $dv / dr = 0$, $d^2v / dr^2 < 0$, must be satisfied. That is

$$k^* = A_o \tag{5.7'}$$

from where, according to (5.1')

$$r^* = r(P^*) = R_o \exp \left(\frac{k_o - A_o}{A_o} \right) \tag{5.8}$$

or else

$$r^* = \exp \left(\frac{B_o - A_o}{A_o} \right) \tag{5.8'}$$

Therefore, for the interval (R_o, R_o') to contain a point with an extreme velocity, it must be true that

$$R_o \geq r^* = \exp \left(\frac{B_o - A_o}{A_o} \right) \geq R_o' \tag{5.9}$$

On the other hand, the second derivative $d^2v/dr^2 = -A_o/r$ demonstrates to us that the said relative extreme, if it exists, can also be a point of maximum velocity.

6. WAVES ACROSS A STRATUM (R_o, R_o') REFLECTED AT THE SURFACE OF RADIUS R_o'.

Up to now we have the formulation corresponding to trajectories that pass through a stratum of radii (R_o, R_o') and constants (A_o, B_o), in accordance to the law of refraction.

Let us now see the formulation that is followed in the case of trajectories whose waves are reflected on the surface of radius R_o'.

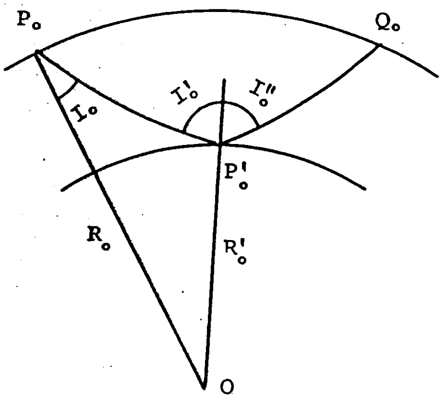


Fig. 6.1

Likewise, let us consider a ray that leaves from a point $P_o = P(R_o)$ with an initial angle I_o and arrives at a point $P_o' = P(R_o')$ with an angle of incidence I_o'.

If this ray reflected on the surface of a radius R_o', *it will return towards a point* Q_o *of the surface of a radius* R_o *with an angle* $I_o'' = I_o'$ *(Fig. 6.1).*

According to this, the equations that determine the trajectory are given by the formulas (4.2), (4.6) and (4.8) that we shall write in the form

$$\sin I_o' = (k_o'/k_o) \sin I_o \tag{6.1}$$

$$\cos I_o' = \cos I_o - (\delta_o/k_o) \sin I_o \tag{6.2}$$

$$\tan (I_o'/2) = e^{\tau_o} \tan (I_o/2) \tag{6.3}$$

being $\delta_o = A_o \Delta(P_o, P_o')$, $\tau_o = A_o T(P_o, P_o')$ (6.

Squaring (6.1) and (6.2), adding and cancelling the common term k_o^2 on both sides, we obtain

$$(k_o'^2 - k_o^2 + \delta_o^2)\sin^2 I_o = 2\,k_o\,\delta_o \sin I_o \cos I_o$$

and for $I_o \neq 0$, we may write

$$\tan I_o = \frac{2\,k_o\,\delta_o}{k_o'^2 - k_o^2 + \delta_o^2} \tag{6.5}$$

Analogously, substituting the values of $\sin I_o'$, $\cos I_o'$ given by (6.1) and (6.2), in the equality (6.3), we shall have

$$e^{\tau_o}\tan(I_o/2) = \frac{\sin I_o'}{1 + \cos I_o'} = \frac{k_o' \tan(I_o/2)}{k_o - \delta_o \tan(I_o/2)}$$

and finally, for $I_o \neq 0$

$$\tan(I_o/2) = \frac{k_o\,e^{\tau_o} - k_o'}{\delta_o\,e^{\tau_o}} \tag{6.6}$$

The elimination of I_o between (6.5) and (6.6) leads us to the equality

$$(k_o\,e^{\tau_o} - k_o')\,(k_o'\,e^{\tau_o} - k_o) = \delta_o^2\,e^{\tau_o} \tag{6.7}$$

which relates the times $T(P_o, P_o')$ to the epicentral distances $\Delta(P_o, P_o')$ through the quantities τ_o and δ_o.

The formula (6.7) can also be written in the form

$$k_o\,k_o'\,(1 + e^{2\tau_o}) = (k_o^2 + k_o'^2 + \delta_o^2)\,e^{\tau_o}$$

from where the following relationship is easily obtained

$$k_o' = k_o \operatorname{Cosh}\tau_o \pm \sqrt{k_o^2 \operatorname{Sinh}^2\tau_o - \delta_o^2} \tag{6.8}$$

that gives us the value k_o' in function of k_o and the data δ_o, τ_o, supplied by tables.

We should remember that in these expressions the case of $I_o = 0$, has been excluded; nevertheless the formula (6.3) shows us that, for $I_o = 0$, it is $I'_o = 0$, and besides

$$k'_o = k_o\, e^{\tau_o} \qquad (6.9)$$

since (6.3), with the help of (6.1), one can write

$$e^{\tau} = \frac{k'_o\,(\,1 + \cos I_o)}{k_o\,(\,1 + \cos I'_o\,)}$$

and the right-hand has the limit (k'_o/k_o) for $I_o = 0$.

We should observe, likewise, that the equality (6.6) also proves (6.9) for $I_o = 0$, for which reason the formula (6.8) with the positive sign is valid in any case, so long as $\delta_o \leq k_o \operatorname{Sinh} \tau_o$

A different version of the equality (6.8) can be obtained in the following way: Discovering the radical and squaring it, after simplifying, we obtain

$$k_o'^2 + k_o^2 - 2\, k_o\, k'_o \operatorname{Cosh} \tau_o = -\,\delta_o^2 \qquad (6.10)$$

and hence, if we make

$$x(\delta_o) = (k_o^2 + k_o'^2 + \delta_o^2)\,/2\, k_o\, k'_o = \operatorname{Cosh} \tau_o$$

the inverse function of $\operatorname{Cosh} \tau_o$ it allows us to write

$$\tau_o = \log\left(\,x + \sqrt{x^2 - 1}\,\right) \qquad (6.11)$$

as an equivalent formula to (6.8) and (6.10).

We should note that for $I'_o = \pi/2$, the waves are tangent to the surface of radius R'_o and therefore must coincide with the waves defined by (5.3) and (5.4) from which the relationships (5.6) was obtained.

In fact, the formulas (6.2) and (6.3), for $I'_o = \pi/2$, give us

$$\tan I_o = k_o/\delta_o \qquad\qquad \tan\,(I_o/2) = e^{-\tau_o}$$

from where we obtain

$$\delta_o = k_o \text{ Sinh } \tau_o$$

In this concrete case, because δ_o and τ_o coincide with $\delta^{(m)}$ and $\tau^{(m)}$, the equality (6.8) is reduced leaving

$$k_o' = k_o \text{ Cosh } \tau_o = \frac{\delta_o \text{ Cosh } \tau_o}{\text{Sinh } \tau_o} = \frac{\delta_o}{\text{Tanh } \tau_o} \qquad (6.12)$$

and eliminating τ_o by means of the equality:

$$\text{Cosh}^2\tau_o - \text{Sinh}^2\tau_o = 1$$

we have

$$k_o^2 \text{ Cosh}^2\tau_o - k_o^2 \text{ Sinh}^2\tau_o = k_o'^2 - \delta_o^2$$

and finally

$$k_o'^2 = k_o^2 + \delta_o^2 \qquad (6.13)$$

The formulas of paragraphs 4, 5 and 6 constitute the foundation for all of our formulation, that will be generalised in successive paragraphs.

7. GENERAL FORMULATION.

The formulas established in the preceding paragraphs can be generalised now. For example, the law of refraction extended to the whole of the interior of the Earth is translated now into the following chain of equalities

$$p_0 = \frac{\sin I_0}{k_0} = \frac{\sin I_0'}{k_0'} = \frac{\sin I_1}{k_1} = \frac{\sin I_1'}{k_1'} =$$

$$= \frac{\sin I_2}{k_2} = \frac{\sin I_2'}{k_2'} = \frac{\sin I_3}{k_3} = \frac{\sin I_3'}{k_3'} =$$

$$= \frac{\sin I_c}{k_c} = \frac{r \sin I}{v_c} \qquad (7.1)$$

In an analogous way, the epicentral distance determined by an incident (or emergent) trajectory across a stratum E_i , according to (4.6), will be in general

$$\Delta_i'^{(m)} = \frac{\cos I_i - \cos I_i'}{A_i\, p_0} \qquad (7.2)$$

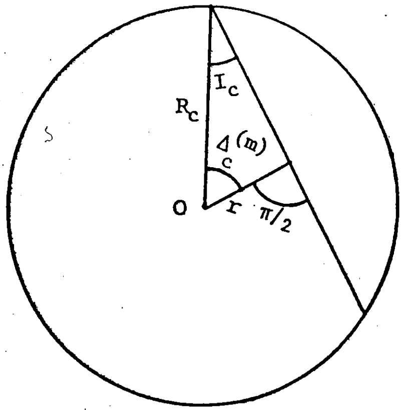


Fig. 7.1

In the particular case of a trajectory across the stratum E_c, we shall have in an isolated way (Fig. 7.1)

$$\Delta_c^{(m)} = \pi/2 - I_c \qquad (7.3)$$

As a particular case, if the trajectory across a stratum E_i , is all contained in the said stratum, with a middle point P_m for which $I_m = \pi/2$, we shall write

$$\Delta_i^{(m)} = \frac{\cos I_i}{A_i\, p_0} \qquad (7.4)$$

In any case, the incident and emergent trajectories across a certain stratum E_i are symmetrical and determine a total epicentral distance

$$\Delta_i' = 2\,\Delta_i'^{(m)} \quad , \quad \Delta_i = 2\,\Delta_i^{(m)} \quad , \quad \Delta_c = 2\,\Delta_c^{(m)} \qquad (7.5)$$

Summarizing, the incident trajectory of a ray that passes through all the stratum determines an epicentral distance

$$\Delta^{(m)} = \frac{1}{p_0}\sum_{i=0}^{3}\frac{\cos I_i - \cos I_i'}{A_i} + \frac{\pi}{2} - I_c =$$

$$= \sum_{i=0}^{3}\Delta_i'^{(m)} + \frac{\pi}{2} - I_c \qquad (7.6)$$

and the epicentral distance determined by its total travel (incident and emergent) will be $\Delta = 2\,\Delta^{(m)}$

Likewise, if the trajectory passes through the stratum E_0, $E_1, \ldots$, $E_{\alpha-1}$, E_α, $E_{\alpha-1}, \ldots$, E_0, being $\alpha \leq 3$, we shall have

$$\Delta^{(m)} = \sum_{i=0}^{\alpha-1} \Delta_i'^{(m)} + \Delta_\alpha{}^{(m)} \qquad (7.7)$$

obtaining like before for the total travel $\Delta = 2\,\Delta^{(m)}$.

The formulas that define the travel-times corresponding to the precedent equalities (7.2), (7.3), (7.4), (7.5), (7.6) and (7.7), are deduced from the equality (4.8), and we shall write them as correlative to the preceding ones, assigning to them the same numeration with the additional symbol (').

The said formulas are:

$$T_i'^{(m)} = -\frac{1}{A_i} \log \frac{\tan (I_i/2)}{\tan (I_i'/2)} \qquad (7.2')$$

$$T_c{}^{(m)} = \frac{R_c \cos I_c}{v_c} = \frac{\cos I_c}{k_c} \qquad (7.3')$$

$$T_i{}^{(m)} = -\frac{1}{A_i} \log \tan (I_i/2) \qquad (7.4')$$

$$T_i' = 2\,T_i'^{(m)}\,, \quad T_i = 2\,T_i{}^{(m)}\,, \quad T_c = 2\,T_c{}^{(m)} \qquad (7.5')$$

$$T^{(m)} = \sum_{i=0}^{3} T_i'^{(m)} + \frac{\cos I_c}{k_c} \qquad (7.6')$$

$$T^{(m)} = \sum_{i=0}^{\alpha-1} T_i'^{(m)} + T_\alpha{}^{(m)} \qquad (7.7')$$

With the intention of completing this general formulation, we should remember that in the trajectory across a stratum E_i , we have (7.4) and (7.4'), so that if we put

$$\delta_i{}^{(m)} = A_i\, \Delta_i{}^{(m)} \qquad\qquad \tau_i{}^{(m)} = A_i\, T_i{}^{(m)}$$

and eliminating between both the angle I_i , we have

$$\delta_i{}^{(m)} = k_i \operatorname{Sinh} \tau_i{}^{(m)} \qquad (7.8)$$

just like we have obtained in (5.6), although now it has a more general character.

In the same way, the condition (5.7), necessary for the existence of a maximum velocity v_i^* in the interior of the stratum E_i, will be in general $k_i^* = A_i$, so that

$$k_i \leq k_i^* = A_i \leq k_i' \qquad (7.9)$$

Evidently, the value R_i' that define the lower radius of the stratum E_i, can be obtained by the equality (5.1), that in this case will be given by the formula

$$R_i' = R_i \exp \frac{k_i - k_i'}{A_i} \qquad (7.10)$$

Another question that can be analysed is the study of the variation of the velocity when it passes from a stratum E_i to the following E_{i+1}.

In fact, if we derive the function v with respect to the depth $z = R_0 - r$, we obtain

$$dv/dz = A - k$$

In the passage from the stratum E_i to E_{i+1} we shall have

$$\left(\frac{dv}{dz}\right)_i = A_i - k_i' \qquad \left(\frac{dv}{dz}\right)_{i+1} = A_{i+1} - k_{i+1}$$

The sign of the terms $A_i - k_i'$ and $A_{i+1} - k_{i+1}$ will show us the increase or decrease of the function velocity v in the passage from the stratum E_i to E_{i+1}.

All of these formulas summarize the main characteristics of the formulation that we shall use in what follows and the main object of our work will consist of determining the constants of each stratum by means of the epicentral distances and the times observed for P and S waves, in accordance with the data that are provided by the tables of the "Bulletin of the Seismological Society of America (1968)", the "Seismological

Tables" by H.Jeffreys and K.E. Bullen (1958), and the "Travel Time Tables for S waves" by M.J. Randall (1971).

To this effect, for each stratum E_i , we shall have to obtain the following elements to define it:

1. Constants A_i , B_i , k_i , and k_i'
2. Radii R_i and R_i' , that limit the said stratum
3. Corresponding depths $z_i = R_0 - R_i$, $z_i' = R_0 - R_i'$
4. Initial angles I_0^i , $I_0'^i$, that determine tangent trajectories to the spheres of radii R_i and R_i' .
5. Initial angle I_0^{i*} of a ray whose trajectory gives rise to a maximum of velocity v_i^* in the interior of the stratum E_i .
6. Radius r_i^* corresponding to the point P_i^* that defines it as point of maximum velocity in the stratum E_i

We should remember, besides, that in the sphere of radius R_i' can be produced a refraction towards the stratum E_{i+1} , or a reflection towards the surface.

In the same way, a tangent ray to the sphere of radius R_i' can return towards the surface or penetrate at a refraction limit angle towards the interior of the stratum E_{i+1} .

Finally, we should point out that the notations used until now refer to P waves, whereas if we refer to S waves we shall use an additional subscript s. In this way, the elements that we have just described will be written: A_{is} , B_{is} , R_{is} , R_{is}' , z_{is} , z_{is}' , I_{0s}^i , $I_{0s}'^i$, I_{0s}^{i*} , v_{is}^* , r_{is}^* , etc.

In essence, the determination of the constants that define a stratum can be obtained by different methods based on the observations, that we can outline in the following way:

1. Approximation of constants by the method of last squares.
2. Calculation of constants through waves reflected at a stratum.
3. Calculation of constants by means of well defined caustic points.

As we shall see next, the calculation by one or another method will depend on how much we can trust in its employment, according to whether we are dealing with waves that pass through the mantle (upper or lower), the outer core and the transition zone or inner core, that is to whether we are dealing with waves P, PP, ..., S, SS, ..., PcP, ScS, PcS, ScP, ... PKP, PKiKP, PKIKP, etc.

8. APPROXIMATION OF CONSTANTS BY THE METHOD OF LEAST SQUARES. TRAJECTORIES IN THE MANTLE.

This method of calculation, that will be applied to the two sub-regions of the mantle, consists of improving previous values for the constants (A_0,k_0) and (A_1,k_1) by the iterative application of a process of least squares, that permits the obtention of corrections $(\delta A_0,\delta k_0)$, $(\delta A_1,\delta k_1)$, with the help of which one achieves a greater concordance with the data observed for Δ and T that are supplied by tables.

To this end, we should remember once again that we have supposed the mantle to be divided into two subregions (upper mantle and lower mantle), of constants respectively (A_0,B_0) , (A_1,B_1), and that are limited by radii (R_0,R_0'), (R_1,R_1'), which are essential to define.

Likewise, it has been supposed that the equalities are verified

$$R_0' = R_1$$

$$k_0 = B_0 - A_0 \log R_0$$

$$k_0' = B_0 - A_0 \log R_0' = B_1 - A_1 \log R_1 = k_1$$

from where

$$B_0 - B_1 = (A_0 - A_1) \log R_1$$

$$I_0' = I_1$$

In accordance with this, we shall consider the development of the process of least squares in two cases:

a) Approximation of the constants A_0, k_0, of the upper mantle.

In this case, putting $\delta_0 = \frac{1}{2} A_0 \Delta_0$, $\tau_0 = \frac{1}{2} A_0 T_0$, we have a relation-ship between δ_0 and τ_0 , given by the formula (5.6), i.e.

$$\delta_0 = k_0 \operatorname{Sinh} \tau_0$$

Supposing then that we have available some previous values A_0^o, k_0^o, and a set of data $\Delta_0^{(i)}$, $T_0^{(i)}$ given by tables, whether for P or S waves, we can calculate a set of values $\delta_0^{(i)} = \frac{1}{2} A_0^o \Delta_0^{(i)}$, $\tau_0^{(i)} = \frac{1}{2} A_0^o T_0^{(i)}$, that taken to the function

$$y^{(i)} = k_0^o \operatorname{Sinh} \tau_0^{(i)} - \delta_0^{(i)}$$

will give, in general, the results $y^{(i)} \neq 0$.

If we now suppose that A_0^* and k_0^* are more probable values of the constants A_0, k_0, we can develop the function $y^{(i)}$ in the neighbourhood of (A_0^o, k_0^o), obtaining

$$y^{(i)} = k_0^o \operatorname{Sinh} \tau_0^{(i)} - \delta_0^{(i)} + \left[\frac{\partial y^{(i)}}{\partial A_0}\right]^o \delta A_0^o + \left[\frac{\partial y^{(i)}}{\partial k_0}\right]^o \delta k_0^o + \dots = 0$$

where the partial derivatives and the independent terms are calculated by the expressions

$$a^{(i)} = \left[\frac{\partial y^{(i)}}{\partial A_0}\right]^o = \frac{1}{A_0^o} \left(k_0^o \, \tau_0^{(i)} \operatorname{Cosh} \tau_0^{(i)} - \delta_0^{(i)}\right)$$

$$b^{(i)} = \left[\frac{\partial y^{(i)}}{\partial k_0}\right]^o = \operatorname{Sinh} \tau_0^{(i)} \qquad (8.2)$$

$$c^{(i)} = \delta_0^{(i)} - k_0^o \operatorname{Sinh} \tau_0^{(i)}$$

According to this, we shall have a set of equations

$$a^{(i)} \, \delta A_0^o + b^{(i)} \, \delta k_0^o = c^{(i)} \qquad (8.3)$$

that can be treated by least squares and lead us to a system of normal equations

$$[11]\,\delta A_0^o + [12]\,\delta k_0^o = c_1$$

$$[21]\,\delta A_0^o + [22]\,\delta k_0^o = c_2 \qquad (8.4)$$

where

$$[11] = \sum_i (a^{(i)})^2 \qquad [22] = \sum_i (b^{(i)})^2$$

$$[12] = [21] = \sum_i a^{(i)}\, b^{(i)} \qquad (8.5)$$

$$c_1 = \sum_i a^{(i)}\, c^{(i)} \qquad c_2 = \sum_i b^{(i)}\, c^{(i)}$$

Once the system (8.4) is resolved, we shall obtain more probable values of the constants A_0, k_0, by means of the equalities

$$A_0^* = A_0^o + \delta A_0^o \qquad k_0^* = k_0^o + \delta k_0^o$$

The process is evidently iterative, being possible the repetition of it with the new values A_0^*, k_0^*, as initial values and continuing in the same way until the differences δA_0, δk_0, are inferior to a prefixed number.

This process has been applied to a set of epicentral distances included in the interval (0°, 40°) and taken from 0,5 in 0,5 degrees.

Specifically, in the case of P waves $A_0^o = 5{,}9 \times 10^{-3}$ and $k_0^o = 1{,}2 \times 10^{-3}$ have been considered as initial data, extracting the data (Δ_0, T_0) from the "Bulletin of the Seismological Society of America (1968)".

In a similar way, in the case of S waves, the initial values considered have been $A_{0s}^o = 3{,}4 \times 10^{-3}$, $k_{0s}^o = 6{,}2 \times 10^{-4}$ and the data (Δ_{0s}, T_{0s}) have been extracted from "Travel Time Tables for S waves (M.J. Randall, 1971)".

The choice of the interval (0°, 40°) has been suggested to us by inspection of the mentioned tables, although for other intervals (2°, 28° ; 5°, 35° ; 1°, 35° , etc.) similar results have been obtained.

The corresponding approximations by least squares in both processes of calculation have led us to the following results:

For P waves

$$A_0 = 5{,}91557395629 \times 10^{-3}$$
$$k_0 = 1{,}16102573176 \times 10^{-3}$$

For S waves

$$A_{0s} = 3{,}45919016463 \times 10^{-3}$$
$$k_{0s} = 6{,}22637501038 \times 10^{-4}$$

In both cases the value for B_0 has been obtained by the relationship $B_0 = k_0 + A_0 \log R_0$, corresponding to the Earth's surface, giving

$$B_0 = 5{,}29785911433 \times 10^{-2}$$
$$B_{0s} = 3{,}09234694999 \times 10^{-2}$$

According to these values of A_0, k_0, in P and S waves, the tables I (for P waves) and II (for S waves) have been constructed, that reflect the relationship that exists between the epicentral distances Δ_0 of the interval (0°, 40°) and the corresponding travel-times T_0.

In both tables are also included the angles I_0 , the times T_t given by the mentioned tables, as well as the differences $T - T_t$, where it can be appreciated that, excluding the most superficial waves (Δ_0 = 1°, 2°, 3°, 4°), the remaining differences $T - T_t$ are, in general, below 2 seconds.

b) Approximation of the constants A_1, k_1, of the lower mantle.

Once the constants A_0, k_0 of the upper mantle have been determined, let us see how the values of the constants A_1, k_1, of the lower mantle can be approximated.

For this, we should remember that in accordance with the notations that we have been using and the relationships $k_0' = k_1$, $I_0' = I_1$, that we established at the beginning of this paragraph, the epicentral distances and the travel-times of each trajectory are: (Δ_0, T_0) for the upper mantle and (Δ_1, T_1) for the lower mantle. Therefore, if we put $\Delta = \Delta_0 + \Delta_1$ and $T = T_0 + T_1$, and we take into account the formulas (7.7) and (7.7'), we shall have

$$\frac{1}{2}\Delta = \frac{1}{p_0}\left(\frac{1}{A_0}\cos I_0 + \left(\frac{1}{A_1} - \frac{1}{A_0}\right)\cos I_1\right)$$

$$\frac{1}{2}T = -\frac{1}{A_0}\log\tan\left(\frac{I_0}{2}\right) - \left(\frac{1}{A_1} - \frac{1}{A_0}\right)\log\tan\left(\frac{I_1}{2}\right)$$

Therefore, if we introduce the notations

$$A = \frac{A_0 - A_1}{A_1} \qquad k^2 = k_1^2 - k_0^2$$

$$\delta = \frac{1}{2}A_0\Delta \qquad \tau = \frac{1}{2}A_0 T \qquad (8.6)$$

the equations that define the trajectory across the whole mantle are the following

$$k_1 \sin I_0 = k_0 \sin I_1 \qquad (8.7)$$

$$\delta = \frac{k_0(\cos I_0 + A\cos I_1)}{\sin I_0} \qquad (8.8)$$

$$\tau = -\log\tan(I_0/2) - A\log\tan(I_1/2) \qquad (8.9)$$

Squaring the equation (8.8) written in the form

$$\delta \sin I_0 - k_0 \cos I_0 = k_0 A \sqrt{1 - \frac{k_1^2}{k_0^2} \sin^2 I_0}$$

one obtains, after dividing by $\cos^2 I_0$

$$(\delta^2 + A^2 k^2) \tan^2 I_0 - 2\delta\, k_0 \tan I_0 + k_0^2 (1 - A^2) = 0 \qquad (8.10)$$

and putting

$$D^2 = \delta^2 + k^2 (A^2 - 1) \qquad (8.11)$$

finally gives

$$\tan I_0 = k_0 \frac{\delta \pm AD}{D^2 + k^2} \qquad (8.12)$$

In all that follows we shall only consider the sign (+) preceding the term AD.

Evidently, the angle I_1 can be calculated by means of the law of refraction (8.7), as a function of the parameters k_0, k_1, and the angle I_0.

An analogous process to the one we have described for the upper mantle can now be applied to the equation (8.9) for values $\delta^{(i)}$, $\tau^{(i)}$, given by tables.

In fact, departing from some initial values A^o, k^o, and from the constants A_0, k_0, already obtained for each value of $\delta^{(i)}$, we shall successively calculate the corresponding values of D, by (8.11), I_0 by (8.12), I_1 by (8.7) and $\tau^{(i)}$ by (8.9).

These values taken to the function

$$y^{(i)} = \tau^{(i)} + \log \tan (I_0/2) + A^o \log \tan(I_1/2) \qquad (8.13)$$

will in general give $y^{(i)} \neq 0$. Therefore, if we develop this function in the neighbourhood of (A^o, k^o) and suppose that $A^* = A^o + \delta A^o$, and $k^* = k^o + \delta k^o$, are more probable values of the said constants, we will be able to calculate the increments $\delta A^o, \delta k^o$, through a process of least squares applied to the equations

$$\left[\frac{\partial y^{(i)}}{\partial A} \right]^o \delta A^o + \left[\frac{\partial y^{(i)}}{\partial k} \right]^o \delta k^o + y^{(i)} = 0$$

for which we shall have to calculate the partial derivatives of the function

$$y = \tau + \log \tan (l_0/2) + A \log \tan(l_1/2) \qquad (8.14)$$

remembering that δ and τ depend on A, but not on k.

In this way, differentiating the equation (8.10) with respect to A and k, we obtain, after some simplifications, the derivatives

$$\frac{\partial l_0}{\partial A} = \frac{k_0}{D} \cos^2 l_1$$

$$\frac{\partial l_0}{\partial k} = - \frac{k\ A}{k_0\ D} \sin^2 l_0 \qquad (8.15)$$

Analogously, the differentiation of the equality (8.7), taking into account (8.15), gives

$$\frac{\partial l_1}{\partial A} = \frac{k_1}{D} \cos l_0 \cos l_1$$

$$\frac{\partial l_1}{\partial k} = \frac{k \sin l_0}{k_0 \cos l_1} \left[\frac{1}{k_1} - \frac{A\ k_1 \sin 2l_0}{2\ k_0\ D} \right] \qquad (8.16)$$

Applying the partial derivation to the function (8.14) with respect to A and k, and bearing in mind that

$$y = y \left[l_0(A,k),\ l_1(A,k);\ A,\ k \right]$$

we obtain

$$\frac{\partial y}{\partial A} = \frac{1}{\sin l_0} \frac{\partial l_0}{\partial A} + \frac{A}{\sin l_1} \frac{\partial l_1}{\partial A} + \log \tan (l_1/2)$$

$$\frac{\partial y}{\partial k} = \frac{1}{\sin l_0} \frac{\partial l_0}{\partial k} + \frac{A}{\sin l_1} \frac{\partial l_1}{\partial k} \qquad (8.17)$$

Hence

$$\frac{\partial y}{\partial A} = \log \tan (I_1/2) + \frac{k_0 \cos I_1}{D \sin I_0} (A \cos I_0 + \cos I_1)$$

$$\frac{\partial y}{\partial k} = \frac{A\,k}{k_1 \cos I_1} \left[\frac{1}{k_1} - \frac{\sin I_1}{D} (A \cos I_0 + \cos I_1) \right] \tag{8.18}$$

In this way, we can repeat the reasoning given for the upper mantle, approximating by the least squares method the values of A and k for P and S waves.

The process has been applied to epicentral distances of the interval (40°, 100°), at intervals 0°.5, for P and S waves.

In the case of the P waves, we have departed from the initial constants $A^o = 0{,}46$, $k^o = 1{,}6 \times 10^{-3}$, extracting the data Δ and T from the tables of the "Bulletin (1968)".

In S waves, the initial values have been $A^o = 0{,}73$, $k^o = 9{,}5 \times 10^{-4}$, extracting the data from the tables "Travel Times (Randall, 1971)".

The definitive results obtained in both cases have been the following:

For P waves

$A = 0{,}471560234314$ $\qquad$ $k = 1{,}63297592077 \times 10^{-3}$

$A_1 = 4{,}019933277870 \times 10^{-3}$ $\qquad$ $k_1 = 2{,}00364445639 \times 10^{-3}$

For S waves

$A = 0{,}73702068772$ $\qquad$ $k = 9{,}53627463370 \times 10^{-4}$

$A_{1S} = 1{,}99145018196 \times 10^{-3}$ $\qquad$ $k_{1S} = 1{,}13889542829 \times 10^{-3}$

c) Calculation of the radius R_1.

The calculation of the radius R_1, that separates the two regions of the mantle, can be obtained by means of the formula (5.1'), that in this case will be

$$R_1 = R_0 \exp\left[\frac{k_0 - k_1}{A_0}\right] \qquad (8.19)$$

The results obtained for P and S waves, are the following

$$R_1 = 5525{,}2036 \text{ Km.}$$

$$R_{1s} = 5487{,}7504 \text{ Km.}$$

Once R_1 is known, the constants B_1 and B_{1s} can be calculated, giving

$$B_1 = 3{,}66437125273 \times 10^{-2}$$

$$B_{1s} = 1{,}82858265398 \times 10^{-2}$$

The ostensible difference $R_1 - R_{1s} = 37{,}5$ Km., that is given for both kinds of waves, is probably motivated by the extraction of data; that has been taken from different tables. It is enough to observe, for example, that for an epicentral distance $\Delta = 100°$, the corresponding times T. supplied by the seismological tables of the "Bulletin (1968)" and of Jeffreys (1956), are, respectively, for P waves

$$T = 13^m\ 46^s.73 \qquad\qquad T = 13^m\ 48^s.4$$

while for S waves the tables of Randall (1971) and of Jeffreys (1956), are

$$T = 25^m\ 25^s.18 \qquad\qquad T = 25^m\ 20^s.4$$

In spite of this difference, if we calculate the angle that corresponds to tangent waves to the sphere of radius R_1, one has for P waves

$$I_0 = \arc\sin\frac{k_0}{k_1} = 35°.4123573733$$

and therefore

$$\Delta_0^{(m)} = \frac{\cos I_0}{A_0\, p_0} = 15°.8163229803$$

In same way, for S waves, the following values are obtained

$$I_{0s} = 33°.1411149070$$

$$\Delta_{0s}^{(m)} = 15°.7952660242$$

These results indicate that the S waves require a greater depth, although the corresponding epicentral distances are almost equal.

9. PcP AND ScS WAVES. RADIUS OF THE OUTER CORE.

The formulation contained in paragraphs 6 and 7, indicate to us that the rays that pass through the mantle (upper and lower) and are reflected at the surface of the outer core, that is, the PcP and ScS waves, satisfy the fundamental equations

$$p_0 = \frac{\sin I_0}{k_0} = \frac{\sin I_1}{k_1} = \frac{\sin I_1'}{k_1'} \tag{9.1}$$

$$\Delta^{(m)} = \frac{1}{p_0} \sum_{i=0}^{1} \frac{\cos I_i - \cos I_i'}{A_i} \tag{9.2}$$

$$T^{(m)} = - \sum_{i=0}^{1} \frac{1}{A_i} \log \frac{\tan (I_i/2)}{\tan (I_i'/2)} \tag{9.3}$$

According to the equalities (8.6), the equations (9.2) and (9.3) can also be written in the forms

$$\delta = A_0 \Delta^{(m)} = \tag{9.2'}$$

$$= \frac{1}{p_0} [\cos I_0 + A \cos I_1 - (1+A) \cos I_1']$$

$$\tau = A_0 T^{(m)} = \tag{9.3'}$$

$$= - \operatorname{logtan} (I_0/2) - A \operatorname{logtan} (I_1/2) + (1+A) \operatorname{logtan} (I_1'/2)$$

Therefore, for the resolution of the system defined by the equations (9.1), (9.2), and (9.3), or its equivalents (9.1), (9.2') and (9.3'), it is necessary to know the value of k_1', that defines in its turn the radius $R_1' = R_2$, that separates the lower mantle and the outer core.

Let us see, then, how the value of the constant k_1' can be obtained by means of the PcP and ScS waves, and more specifically by means of these waves when $I_0 = 0$, since in this case we estimate that the error committed in its calculation is the smallest possible.

To this end, we can apply the formula (6.9) to the stratum E_0 and E_1, writing the equalities

$$k_0' = k_1 = k_0\, e^{A_0 T_0} \qquad\qquad k_1' = k_1\, e^{A_1 T_1} \tag{9.4}$$

designating T_0 and T_1 the travel-times inverted by the ray in its incident trajectory across E_0 and E_1, respectively. Putting $T^{(m)} = T_0 + T_1$, from the formulas (9.4) we deduce

$$T_0 = \frac{1}{A_0} \log \frac{k_1}{k_0} \tag{9.5}$$

hence

$$k_1' = k_1 \exp \left[\, (T^{(m)} - T_0)\, A_1 \,\right] \tag{9.6}$$

and finally, according to (5.1)

$$R_1' = R_2 = R_1 \exp \frac{k_1 - k_1'}{A_1} \tag{9.7}$$

Applying these equalities to the waves PcP, with $I_0 = 0$, and with the time $T^{(m)} = 255^s.65$, deduced from the seismological tables of the "Bulletin (1968)", one has the following results.

$T_0 = 92^s.2419174582$

$k_1' = 3.86459673672 \times 10^{-3}$

$R_2 = 3477.76192858$ Km.

Analogously, using the data that the Seismological Tables of Jeffreys and Bullen (1956) provide for ScS waves, with $I_0 = 0$ and $T^{(m)}_s = 467^s.85$, the following results are obtained

$$T_{0s} = 174^s.563880921$$

$$k'_{1s} = 2.0423958711 \times 10^{-3}$$

$$R_{2s} = 3486.25714643 \text{ Km.}$$

The small discordance $R_{2s} - R_2 = 8.5$ Km., that exits between both radii, is probably originated by the extraction of data from different tables, and we believe it to be without importance.

Tables for P and S waves

The knowledge of the constants A_0, B_0, k_0, A_1, B_1, k_1, and the values R_0, R_1, R_2, for P and S waves, permits the construction of the tables III (for P waves) and IV (for S waves), that gather together the principal data on the transmission of waves across the whole mantle. Subsequently we shall describe the process followed in the said construction, referring only to the case of P waves, since for S waves it is enough to repeat the same plan of calculation with the corresponding values to S waves, that is to say A_{0s}, B_{0s}, k_{0s}, etc.

Upper mantle

We have calculated the angle $I_0^1 = 35°.4123$, that corresponds to the angle $I'_0 = 90°$, in other words to the tangent ray to the sphere of radius R_1, from which we have deduced the epicentral distance $\Delta_0 = 31°.6326$.

Epicentral distances have been considered, degree by degree, for the interval (0°, 31°.6326), calculating next $\delta = A_0\Delta/2$ and after that $\tau = \text{arg sinh } (\delta/k_0)$, obtaining the values of T by the formula $T = 2\tau/A_0$.

Lower mantle

Once calculated the value I_0^2 = arc sin (k_0/k_1') = 17°.4832 from which Δ = 101°.5901 has been deduced, epicentral distances have been considered, degree by degree, for the interval (31°.6326, 101°.5901), calculating $\delta = A_0\Delta/2$ and subsequently D by the formula (8.11), I_0 by (8.12), I_1 by (8.7), and finally τ by (8.9), from where one obtains $T = 2\tau/A_0$.

The remaining data contained in the tables III and IV, will be defined in the explanation of these tables, at the end of this work.

Tables for PcP and ScS waves

The construction of these tables depends on the resolution of the system of equations (9.1), (9.2'), and (9.3'). In fact, given an angle I_0, the angles I_1 and I_1', can be calculated by means of (9.1) and afterwards δ and τ by the formulas (9.2'), (9.3'). The process can be reversed numerically, in such a way that for each δ, the value I_0 is obtained, and finally one obtains τ.

The relationship that exists between the epicentral distances and the travel-times, for PcP and ScS waves, has been included in the tables V and VI, respectively.

10. PcS AND ScP WAVES

The incident P waves that are reflected at the outer core as S waves, are denominated PcS waves. Correlatively, the incident S waves that are reflected at the outer core as P waves, receive the name of ScP waves.

In accordance with the equations (9.1), (9.2') and (9.3'), that define the PcP and ScS waves for incident trajectories, we can now write the corresponding equations to the PcS and ScP waves, completing the incident trajectories with their corresponding emergent trajectories. So, we shall have

$$p_0 = \frac{\sin I_0}{k_0} = \frac{\sin I_1}{k_1} = \frac{\sin I'_1}{k'_1} =$$

$$= \frac{\sin I'_{1s}}{k'_{1s}} = \frac{\sin I_{1s}}{k_{1s}} = \frac{\sin I_{0s}}{k_{0s}} \qquad (10.1)$$

$$\Delta = \frac{1}{p_0}\left\{ \frac{1}{A_0}\left[\cos I_0 + A \cos I_1 - (1+A)\cos I'_1\right] + \right.$$

$$\left. + \frac{1}{A_{0s}}\left[\cos I_{0s} + A_s \cos I_{1s} - (1+A_s)\cos I'_{1s}\right]\right\}$$

$$(10.2)$$

$$T = \frac{1}{A_0}\left[-\log\tan(I_0/2) - A\log\tan(I_1/2) + (1+A)\log\tan(I'_1/2)\right] +$$

$$+ \frac{1}{A_{0s}}\left[-\log\tan(I_{0s}/2) - A_s\log\tan(I_{1s}/2) + (1+A_s)\log\tan(I'_{1s}/2)\right]$$

$$(10.3)$$

that show the identity of epicentral distances and travel-times by both kinds of waves. We consider a single radius R_2 for the definition of the outer core, since the said radius has an influence on the determination of the constants k'_1 and k'_{1s}.

Taking in both cases the radius R_2, the difference that is obtained in the determination of k'_{1s}, is

$$B_{1s} - A_{1s} \log R_2 - (B_{1s} - A_{1s} \log R_{2s}) = 4.85863446 \times 10^{-6}$$

that affects the sixth decimal of k'_{1s}.

Now then, estimating that a greater security exists in the observation of the P waves, and bearing in mind that the S waves are not transmitted across the outer core, in what follows we shall consider that the radius R_2 define the superior radius of the outer core.

On this assumption, given an initial angle I_0, we will be able to calculate the angles I_1, I'_1; I_{0s}, I_{1s}, I'_{1s}, obtaining the corresponding values of Δ and T. Inverting the process until a given value of Δ is reached, we shall obtain the relationships that exist between the epicentral distances Δ and the travel-times T.

The results of these calculations have been gathered in the table VII.

11. SUMMARY ON THE TRANSMISSION OF WAVES ACROSS THE MANTLE.

In the following tables, we include the most relevant data on the transmission of P and S waves across the mantle (upper and lower). That is, the radii and depths (R_0,R_0'), (z_0,z_0'), (R_1,R_1'), (z_1,z_1'), the constants (A_0,B_0), (A_1,B_1), the quantities (k_0,k_0'), (k_1,k_1'), the velocities (v_0,v_0'), (v_1,v_1'), the extreme incidence angles I_0^1, I_0^2, and finally the epicentral distances Δ_0^1, Δ_0^2 and travel-times T_0^1, T_0^2 corresponding to these extreme angles.

		P WAVES	S WAVES
U	R_0	6371.0280 km.	6371.0280 km.
P	z_0	0.0000 km.	0.0000 km.
P	R_0'	5525.2036 km.	5487.7504 km.
E	z_0'	845.8244 km.	883.2776 km.
R	A_0	$5.91557395629 \times 10^{-3}$	$3.45919016463 \times 10^{-3}$
	B_0	$5.29785911433 \times 10^{-2}$	$3.09234694999 \times 10^{-2}$
M	k_0	$1.16102573176 \times 10^{-3}$	$6.22637501038 \times 10^{-4}$
A	k_0'	$2.00364445639 \times 10^{-3}$	$1.13889542829 \times 10^{-3}$
N	v_0	7.3969 km./sec.	3.9668 km./sec.
T	v_0'	11.0705 km./sec.	6.2500 km./sec.
L	I_0^1	35°.4123573733	33°.1411149070
E	Δ_0^1	31°.6326459606	31°.5905320483
	T_0^1	386.015262446 sec.	700.835279548 sec.

		P WAVES	S WAVES
	R_1	5525.2036 km.	5487.7504 km.
L	z_1	845.8244 km.	883.2776 km.
O	R_1'	3477.7619 km.	3486.2571 km.
W	z_1'	2893.2661 km.	2884.7709 km.
E	A	$4.71560234314 \times 10^{-1}$	$7.37020687721 \times 10^{-1}$
R	A_1	$4.01993327787 \times 10^{-3}$	$1.99145018196 \times 10^{-3}$
	B_1	$3.66437125273 \times 10^{-2}$	$1.82858265398 \times 10^{-2}$
	k	$1.63297592077 \times 10^{-3}$	$9.53627463370 \times 10^{-4}$
M	k_1	$2.00364445639 \times 10^{-3}$	$1.13889542829 \times 10^{-3}$
A	k_1'	$3.86459673672 \times 10^{-3}$	$2.04239587110 \times 10^{-3}$
N	v_1	11.0705 km./sec.	6.2500 km./sec.
T	v_1'	13.4401 km./sec.	7.1203 km./sec.
L	I_0^2	17°.4831975615	17°.7495262236
E	Δ_0^2	101°.590131476	105°.829912683
	T_0^2	836.264013552 sec.	1580.06397046 sec.

Now in addiction to this paragraph on the transmission of waves across the mantle it is convenient to make note of some characteristics of the S waves trajectories, that have not been emphasized sufficiently.

In effect, the condition (5.7) applied to the S wave trajectories in the lower mantle proves that there exists a value k^*_{1s}, such that

$$k_{1s} < k^*_{1s} = A_{1s} < k'_{1s}$$

For such a value the corresponding velocity v^*_{1s} well be maximum and will be produced over a sphere of radius

$$r^*_{1s} = R_{1s} \exp \frac{k_{1s} - k^*_{1s}}{A_{1s}} = 3576.59387525 \text{ Km.}$$

in other words at a depth

$$z^*_{1s} = 2794.4341 \text{ Km.}$$

The value that the velocity reaches at this point, is

$$v^*_{1s} = k^*_{1s}\, r^*_{1s} = 7.12260852366 \text{ Km./sec.}$$

The initial angle I^*_{0s} corresponding to the trajectory tangent to the sphere of radius r^*_{1s} , will be

$$I^*_0 = \text{arc sin } (k_{0s}/k^*_{1s}) = 18^\circ.2193257018$$

from where the following values are deduced

$$\Delta^*_1 = 102^\circ.548379515 \qquad T^*_1 = 1551.66554792 \text{ sec.}$$

As a consequence of this, the trajectories of the S waves across the lower mantle, for which $I_{0s} < I^*_{1s}$, determine that the ray changes its curvature twice.

12. GENERAL CONDITIONS FOR THE EXISTENCE OF RELATIVE MAXIMA AND MINIMA.

The equations that define a trajectory across the stratum E_0, E_1, ..., E_α, being $\alpha \leq 3$, have been developed in paragraph 7 and more specifically in the formulas (7.1), (7.7) and (7.7').

Now then, as a consequence of the law of refraction, contained in (7.1), the angles I_i, I'_i, and the quantities k_i, k'_i, are related to the initial angle I_0 and the initial constant k_0, in such a way that the incident epicentral distance $\Delta^{(m)}$ and the corresponding travel-time $T^{(m)}$ are exclusive functions of the variable I_0, and of a group of constants A_0, A_1, ..., A_α, k_0, k'_0, k_1, k'_1, ..., k_α, through certain functions I'_0, I_1, I'_1, ..., I_α, that depend on the variable I_0 . Then, in both cases, it deals with a function of functions of the variable I_0.

Representing by I_i any of the angles I'_0, I_1, I'_1, ..., I_α, and by k_i any of the constants k_0, k'_0, k_1, k'_1, ..., k_α, the formula (7.1) can be written in the form

$$\frac{\sin I_0}{k_0} = \frac{\sin I_i}{k_i} \tag{12.1}$$

Consequently, if we derive this relationship with respect to I_0, we shall have

$$\frac{\cos I_0}{k_0} = \frac{\cos I_i}{k_i} \frac{dI_i}{dI_0} \tag{12.2}$$

and eliminating the constants k_0, k_i, by means of the formula (12.1), we obtain the general formula

$$\frac{dI_i}{dI_0} = \frac{\tan I_i}{\tan I_0} \tag{12.3}$$

Introducing the functions

$$\Phi = \sum_{i=0}^{\alpha-1} \frac{1}{A_i} \left(\frac{1}{\cos I_i'} - \frac{1}{\cos I_i} \right) - \frac{1}{A_\alpha \cos I_\alpha} \tag{12.4}$$

the derivatives of the functions $\Delta^{(m)}$ and $T^{(m)}$ with respect to I_0, taking in account (12.3), and once simplified, are

$$\frac{d\Delta^{(m)}}{dI_0} = \frac{\Phi}{p_0 \tan I_0}$$

$$\frac{dT^{(m)}}{dI_0} = \frac{\Phi}{\tan I_0} \tag{12.5}$$

Dividing both equalities we have the general relationship

$$p_0\, d\Delta^{(m)} = dT^{(m)} \tag{12.6}$$

valid for any trajectory, independently of the number of stratum crossed. On the other hand, given that the emergent trajectory is symmetrical to the incident trajectory, the formula (12.6) is also applicable to the total epicentral distance Δ and to the total travel-time T. Consequently, we also have

$$p_0\, d\Delta = dT \tag{12.7}$$

an equality that coincides with the one obtained in (1.2).

Conditions for the existence of relative extremes

According to (12.5), the necessary condition for the existence of a relative maximum or minimum of the functions $\Delta^{(m)}$ and $T^{(m)}$, is

$$\Phi = 0 \tag{12.8}$$

This condition will have to be complemented by the value that the second derivatives adopt, in order to be able to specify the true character of the relative extreme.

So, introducing a new function

$$\Psi = \sum_{i=0}^{\alpha-1} \frac{1}{A_i} \left[\frac{1}{\cos^3 l_i'} - \frac{1}{\cos^3 l_i} \right] - \frac{1}{A_\alpha \cos^3 l_\alpha} \tag{12.9}$$

after some calculations and simplifications, one obtains

$$\frac{d^2T^{(m)}}{dl_0^2} = \frac{\Psi}{\tan^2 l_0} \tag{12.10}$$

Besides, deriving (12.6) with the condition $\Phi = 0$, we obtain

$$\frac{d^2\Delta^{(m)}}{dl_0^2} = \frac{1}{p_0} \frac{d^2T^{(m)}}{dl_0^2} \tag{12.11}$$

and consequently both second derivatives have the same sign, since $p_0 > 0$.

It will be enough, then, to examine the sign of the function Ψ, to see if the annulation of the function Φ leads to a maximum, a minimum or to an expression that requires the calculation of derivatives of a higher order.

Putting

$$\nu_\alpha = \sum_{i=0}^{\alpha-1} \frac{1}{A_i} \left[\frac{1}{\cos l_i'} - \frac{1}{\cos l_i} \right] \tag{12.12}$$

$$\mu_\alpha = \sum_{i=0}^{\alpha-1} \frac{1}{A_i} [\cos I_i - \cos I_i'] \qquad (12.13)$$

the condition (12.8) for the existence of a relative extreme of the epicentral distance $\Delta^{(m)*}$, allows us to obtain

$$\frac{1}{A_\alpha} = \nu_\alpha \cos I_\alpha \qquad (12.14)$$

and taking this equality to the formula (7.7) we shall have

$$\Delta^{(m)*} = \frac{1}{p_0} (\mu_\alpha + \nu_\alpha \cos^2 I_\alpha) \qquad (12.15)$$

from where

$$\cos^2 I_\alpha = \frac{\Delta^{(m)*} p_0 - \mu_\alpha}{\nu_\alpha} \qquad (12.16)$$

Summarizing this process, let us suppose we know the constants A_0, A_1, ..., $A_{\alpha-1}$, k_0, k_0', ..., $k_{\alpha-1}$, $k_{\alpha-1}'$, and a relative extreme for an epicentral distance $\Delta^{(m)*}$ and the corresponding travel-time $T^{(m)*}$, both extracted from tables. Then, for each initial angle I_0, we can calculate the angles I_0', I_1, ..., $I_{\alpha-1}'$, by means of the formula (12.1) and quantities ν_α, μ_α, through (12.12) and (12.13), obtaining the angle I_α by the equality (12.16) and the value of A_α according to (12.14). Finally, the equality (7.7') will give us the corresponding value of $T^{(m)}$.

These calculations will have to be repeated for different values of I_0 until an angle I_0^* is found, whose travel-time $T^{(m)}$ verifies the equality $T^{(m)} = T^{(m)*}$.

The values of I_α, A_α, corresponding to the initial angle I_0^*, and the formulas

$$k_\alpha = \frac{k_0 \sin I_\alpha}{\sin I_0^*} \qquad B_\alpha = k_\alpha + A_\alpha \log R_\alpha$$

allow us to obtain the constants A_α, B_α, k_α, that define the stratum E_α.

The function Ψ defined in (12.9), will tell us the true character of this relative extreme.

The previous process for the calculation of relative extremes when $\alpha \leq 3$, can be repeated for all the stratum E_0, E_1, E_2, E_3, E_c. In this case the fundamental formulas (12.1) and (12.3) continue to be valid for $I_i = I_c$, and the epicentral distance $\Delta^{(m)}$ and the travel-time $T^{(m)}$ are given by the formulas (7.6) and (7.6').

Now, making

$$\Phi_c = \frac{\nu_4}{p_0} - \tan I_c \tag{12.17}$$

the derivatives of $\Delta^{(m)}$ and $T^{(m)}$ with respect to I_0, adopt the forms

$$\frac{d\Delta^{(m)}}{dI_0} = \frac{\Phi_c}{\tan I_0} \tag{12.18}$$

$$\frac{dT^{(m)}}{dI_0} = \frac{\Phi_c\, p_0}{\tan I_0} \tag{12.19}$$

from which the following formula is derived

$$p_0\, d\Delta^{(m)} = dT^{(m)}$$

that coincides with (12.6), as was to be expected.

Consequently, the relative extremes, given by the condition $\Phi_c = 0$, determine I_c by means of the equality

$$\tan I_c = \frac{\nu_4}{p_0} \tag{12.20}$$

and k_c through the formula

$$k_c = \frac{\sin I_c}{p_0} \tag{12.21}$$

making possible, for each I_0, the calculation of the angles I_0', I_1, I_1', I_2, I_2', I_3, I_3', through the equalities (12.1), the angle I_c by means of (12.20), and finally k_c through (12.21).

Agreeing with this, given an observed relative extreme, which relates the quantities $\Delta^{(m)*}$ and $T^{(m)*}$, one can obtain numerically the corresponding initial angle I_0^*, which determines I_c, k_c, for us.

Calculating the second derivatives for $\Phi_c = 0$, the equality (12.11) is reproduced, and the sign of the expression

$$\Psi_c = \sum_{i=0}^{3} \frac{1}{A_i} \left(\frac{1}{\cos^3 I_i'} - \frac{1}{\cos^3 I_i} \right) - \frac{\sin^2 I_c}{k_c \cos^3 I_c} - 2\nu_4$$

indicates to us whether we are dealing with a minimum ($\Psi_c > 0$), a maximum ($\Psi_c < 0$) or with an expression ($\Psi_c = 0$) that requires the calculation of higher order derivatives.

13. PKP WAVES

The trajectories which after passing through the mantle penetrate the outer core receive the name of PKP waves and the equations which define their trajectories respond to the general formulation developed in paragraph 7, and in a more particular way to the formulas (7.1), (7.7), and (7.7') with $\alpha = 2$.

Therefore, it is necessary to calculate the values of the fundamental constants A_2, B_2, k_2, that determine the outer core.

According to our developments, we can follow processes of comparisons of results (Δ,T) given by tables or else choose some condition of relative extreme as we have seen in previous paragraphs.

A profound analysis of the results obtained with these criterion has lead us to use the condition of relative minimum of the point B-cusp., frequently observed and which according to the tables of the Bull. (1968) exists for an epicentral distance $\Delta = 143°$ and a travel-time $T = 1171.5$ sec., which figures in brackets as if it were extrapolated.

On the other hand, as K.E. Bullen and B.A. Bolt (1985) point out "the crucial location of the cusps. B, D and C. are still not very exact", for which reason analyzing the results obtained we obtain a greater concordance of travel-times T in the branch AB taking for the point B-cusp. an epicentral distance $\Delta = 144°$ (Qamar, 1973) and a travel-time T = 1172 sec.

So then, considering the point B-cusp. to be a relative minimum of an epicentral distance $\Delta = 144°$ and a travel-time T = 1172 sec., the following results have been obtained:

$I_0 = 14°.9681526307$	$I'_0 = 26°.4701364140$
$I_1 = 26°.4701364140$	$I'_1 = 59°.2850752016$
$I_2 = 28°.5871232641$	
$\nu_2 = 223.000008678$	$\mu_2 = 107.609233227$
$A_2 = 5.10688201126 \times 10^{-3}$	$k_2 = 2.15092122295 \times 10^{-3}$
$B_2 = 4.37931737692 \times 10^{-2}$	$v_2 = 7.48039194055$ Km./sec.

The value $\Psi = 1279.027$ indicates to us that the point B-cusp. evidently corresponds with a well pronounced minimum, since as will be found in table VIII of PKP waves, the energy originated by an initial beam of rays of 1°.7, is concentrated at its exit around a beam of rays of 1°, in an interval of time of some 4 seconds.

We should point out that the ray which is tangent to the outer core at R_2, and which according to the table III, returned to the surface with an epicentral distance $\Delta = 101°.6$, suffers a strong refraction at R_2 on penetrating the outer core, returning to the surface of radius R_0 with an epicentral distance $\Delta = 173°.6$.

This makes us see that the results obtained for the branch AB of the table VIII do not begin with an epicentral distance $\Delta = 180°$ (Bull., 1968) or $\Delta = 188°$ (Gutenberg, 1959) which seems to indicate that the interval (180°, 173°.6) of epicentral distances is not due to refraction phenomenon, but to diffraction phenomenon or to irregularities in the transition from the mantle to the outer core, as is pointed out by T. Lay, T.J. Ahrens, P. Olson, J. Smyth and D. Loper in Physics Today (October, 1990).

As we shall see in the following paragraph, once the constants of the transition zone have been determined, one obtains

$$R_2' = 1428.12773922 \text{ Km.} \qquad k_2' = 6.69617209644 \times 10^{-3}$$

Then, according to this, we can give the following significant results about the transmission of waves through the outer core:

a) Interval of initial angles I_0 for PKP waves.

According to the laws of refraction, the interval (I_0^2, I_0^3) of angles that determine PKP waves are

$$I_0^2 = \text{arc sin} \frac{k_0}{k_1'} = 17°.4831975615$$

$$I_0^3 = \text{arc sin} \frac{k_0}{k_2'} = 9°.98477507077$$

The initial angle I_0^2 determine the values $\Delta = 173°.634$ and $T = 1302.582$ sec., with which table VIII begins. Starting from these initial values, while I_0 diminishes, from 17°.483 to 14°.968, the epicentral distance Δ decreases from 173°.634 to 144°, and the travel-time T from 1302.582 sec. to 1172 sec.

Subsequently, the slow diminishing of I_0, from 14°.968 to 9°.985, gives place to an increase of Δ and T, until they reach the maximum values $\Delta = 172°.522$ and $T = 1258.562$ sec., that could correspond to the point C(Δ = 169°) of Gutenberg.

b) Maximum velocity in the outer core

Taking in account that $k_2 < A_2 < k_2'$ (paragraph 5), a point of maximum velocity v_2^* will exist when the condition (5.7) is fulfilled, i.e.

$$k_2^* = A_2$$

The radius r_2^* of the sphere that contains such points of maximum velocity, will be

$$r_2^* = R_2 \exp \frac{k_2 - k_2^*}{A_2} = 1949.49404522 \text{ Km.}$$

in which v_2^* reaches its maximum value

$$v_2^* = k_2^* \, r_2^* = 9.95583607059 \text{ Km./sec.}$$

The initial angle I_0^* that determines a ray which is at a tangent to the sphere of radius r_2^*, will be

$$I_0^* = \text{arc sin} \frac{k_0}{k_2^*} = 13^\circ.1408299261$$

with which the elements

$$\Delta^* = 148^\circ.012118687 \qquad T^* = 1186.35359102 \text{ sec.}$$

correspond.

c) Possible identification of the branch GH (Bull., 1968) by reflection

Starting at the point of maximum velocity v_2^*, the velocities of the waves PKP continuously decrease with the depth, which constitutes a singular case, if we except those other cases in which there exists a discontinuity of velocities, as occurs in the passage of waves from the lower mantle to the outer core.

In consequence, if we assume that the decrease of the velocities can give rise to reflection phenomenon, we can calculate a table (Δ,T) that will serve to identify it.

To this end, if we assign an epicentral distance $\Delta_t^{(m)}$ given by the table of the branch GH, we shall have

$$\Delta_t^{(m)} = \frac{\mu_3}{p_0}$$

from where

$$I_2'(I_0) = \text{arc cos}\left[A_2\left(\mu_2 - p_0\,\Delta_t^{(m)}\right) + \cos I_2\right]$$

$$k_2'(I_0) = \frac{\sin I_2'}{p_0}$$

$$T^{(m)}(I_0) = -\sum_{i=0}^{2}\frac{1}{A_i}\log\frac{\tan(I_i/2)}{\tan(I_i'/2)}$$

in accordance with our formulation.

Comparing this result of $T^{(m)}(I_0)$ with the time $T_t^{(m)}$ assigned in the tables, we shall obtain numerically the value of I_0 which verifies the condition $T^{(m)}(I_0) = T_t^{(m)}$.

In this way, we have constructed table IX, which presents the following particularities:

1.- The differences $T - T_t$ for values of Δ between 125° and 142°.68 are null.

2.- Above $\Delta = 142°.68$ the reflections at the sphere of radius r_2^* present small errors in $T - T_t$, in such a way that for $\Delta = 145°$, one has $T - T_t = 1.66$ sec., and $I_2' = 88°,5$ on the sphere of radius r_2^*. These results justify the characteristics of the branch GH, whose travel-times T_t are given in brackets for values of Δ higher than 145°.

3.- It should be observed that the depth at which this supposed reflection is produced increases slowly from $\Delta = 142°.68$ to $\Delta = 125°$.

Finally, we shall remember that tables VIII and IX summarize the behaviour of the PKP and PKP* waves through the outer core.

14. TRANSITION ZONE IN THE OUTER CORE.

It is evident that in the interior of the outer core (stratum E_2) no other minimum different to the B-cusp., can exist, since the condition $\Phi = 0$, given by (12.4) determines a single value of I_2 by means of the equality

$$\cos I_2 = \frac{1}{A_2 \nu_2}$$

This suggests the necessity of a transition zone which allows the establishment of the existence of other singular points, which have been indicated by several authors.

Indeed, H. Jeffreys and K.E. Bullen (1958) as well as the tables of the Bull (1968) point out the existence of the point D for $\Delta = 110°$ and $T = 1113$ sec., under whose epicentral distance the results in T have to be extrapolated.

Likewise, Gutenberg (Physics of the Earth's Interior, pag. 104, Acad. Press, 1959), according to M.H.P. Bott (The interior of the Earth, pag 152, Edward Arnole Pub, Ltd. 1971), indicates that "the epicentral distance of the emerging ray progressively decreases from about 188° to 143° and then it increases again to about 169° (point C)."

Finally, G.L. Choy and V.F. Cormier (The structure of the inner core inferred from short-period and broadband GDSN data, Geoph. Journal of the R.A.S., vol. 72, nº 1, 1983) establish the existence of two singular points which denominate D-cusp. and C-cusp., whose respective epicentral distances are $\Delta = 121° \pm 1°$ and $\Delta = 154° \pm 2°$.

The examination of the results that are deduced from such possibilities has made us decide, in principle, to choose as more likely the point D-cusp. (Choy, Cormier) though the concordance of the results for Δ and T are higher taking for D-cusp., the epicentral distance $\Delta = 122°$ and a travel-time $T = 1135{,}9$ sec. which also satisfies the result $121° \pm 1°$, shown by Choy and Cormier, as well as the data from Bull's Tables (1968).

And again, inspection of the tables (Δ,T) suggest that there exists no discontinuity of velocities in the passage from the outer core to the transition zone, for which it seems reasonable to suppose that the condition $k_2' = k_3$ is verified.

So then, imposing the conditions:

a) The point D-cusp. (Δ = 122°, T = 1135.9 sec.) is a relative minimum.

b) $k_2' = k_3$

we have

$$l_2' = l_3$$

$$\Phi = \nu_3 - \frac{1}{A_3 \cos l_3} = 0 \tag{14.1}$$

$$\Delta^{(m)*} = \frac{122^\circ}{2} = \frac{1}{p_0}\left[\mu_3 + \frac{\cos l_3}{A_3}\right]$$

Therefore, if we denote by R and S the known quantities

$$R = \nu_2 - \frac{1}{A_2 \cos l_2} \tag{14.2}$$

$$S = \frac{1}{p_0}\left[\mu_2 + \frac{\cos l_2}{A_2}\right] \tag{14.2'}$$

we shall have

$$R = \left[\frac{1}{A_3} - \frac{1}{A_2}\right]\frac{1}{\cos l_3} \tag{14.3}$$

$$\Delta^{(m)*} - S = \left[\frac{1}{A_3} - \frac{1}{A_2}\right]\frac{\cos l_3}{p_0} \tag{14.3'}$$

from where

$$\cos^2 l_3 = \frac{[\Delta^{(m)*} - S]\, p_0}{R} \tag{14.4}$$

and the formula (14.3) will allow us to obtain

$$A_3 = \frac{A_2}{1 + R\, A_2 \cos I_3} \tag{14.5}$$

Summarizing, given the values of the constants A_0, A_1, A_2, k_0, k_1, k_1', k_2, we can proceed in the following way: For each initial value I_0, we shall successively calculate the angles I_1, I_1', I_2, and the quantities μ_2, ν_2, R and S, as well as I_3 (14.4) and A_3 (14.5). Taking these value to the formula (7.7') with $\alpha = 3$, we shall have a travel-time

$$T^{(m)} = - \sum_{i=0}^{2} \frac{1}{A_i} \log \frac{\tan (I_i/2)}{\tan (I_i'/2)} - \frac{1}{A_3} \log \tan (I_3/2)$$

The process is iterative and is completed when $T^{(m)} = T_t^{(m)} = 567.95$ sec. The results obtained by this process have been the following:

$I_0 = 6°.61707063348$ $\qquad$ $I_0' = 11°.4705347052$

$I_1 = 11°.4705347052$ $\qquad$ $I_1' = 22°.5547180776$

$I_2 = 12°.3264467133$ $\qquad$ $I_2' = 41°.6518460511$

$I_3 = 41°.6518460511$

$R = -182.590441981$ $\qquad$ $S = 2.09175113318$

$A_3 = 1.68397895213 \times 10^{-2}$

$k_3 = k_2' = 6.69617209644 \times 10^{-3}$

$B_3 = 0.129022417085$

$v_3 = v_2' = 9.56298911752$ km./sec.

$R_3 = R_2' = 1428.12773922$ km.

$\Psi = 175.710052877 > 0$

According to what precedes, the waves PKP which penetrate the transition zone, and which we denote by $PKP^{(T)}$ waves, are produced for initial angles smaller than $I_0 = 9°.9848$.

Nevertheless, since we still have not calculated the radius R_c of the inner core, we do not know the angle I_0 that limits the set of rays $PKP^{(T)}$ belonging to the said zone. However, admitting for now the value $R_c = 1216$ Km., which figures in numerous publications as the radius of the inner core we can construct a table of values of I_0, ranging from 9°.9848 to 7°.0919, which determine the pairs of values (Δ,T) corresponding to the waves $PKP^{(T)}$, within the conditions that have been established.

The said table, which we include as table X, provides us with the corresponding results for the interval (172°.5, 122°.5) of values of Δ.

In relation to this; it should be observed that:

a) Taken $R_c = 1216$ Km. , the epicentral distance $\Delta = 122°$ is not attained.

b) The travel-times T_t corresponding to this table do not figure in any of the tables of PKP waves that have been consulted, for which reason it is to be presumed that these waves are not easily identifiable.

c) In the passage of PKP waves from the outer core to $PKP^{(T)}$ waves of the transition zone, a change in velocity is produced in such a way that the decrease of velocities in the outer core (negative acceleration) is transformed into an increase of velocities (positive acceleration) in the transition zone, since, as we saw in paragraph 7, one has

$$A_2 - k_2' = -\ 1.58929008518 \times 10^{-3}$$

$$A_3 - k_3 = 1.01436174249 \times 10^{-2}$$

15. INNER CORE.

For a correct definition of the inner core, it is necessary to determine the radius R_c where the transition zone finishes.

To this end we can make use of the data tabulated for PKP waves, whether in the Bull. (1968) or in the Seismological Tables of H. Jeffreys and K.E. Bullen (1958).

Inspection of the said tables shows us the existence of two singular points which can serve for the determination of R_C. These are the points denominated by E and F. The first corresponds to a ray which penetrates the inner core and appears as first data not extrapolated, i.e. that the travel-times T_t inferior to T_E figure in brackets. The second corresponds to the axial ray.

Depending on the way one or the other table is consulted, the aforementioned points are determined in the following manner:

Tables of H. Jeffreys and K.E. Bullen

Point E ($\Delta_E = 145°$, $T_E = 19^m\ 39^s.2$)
Point F ($\Delta_F = 180°$, $T_F = 20^m\ 12^s.2$)

Tables of the Bulletin (1968)

Point E ($\Delta_E = 146°$, $T_E = 19^m\ 40^s.5$)
Point F ($\Delta_F = 180°$, $T_F = 20^m\ 11^s.0$)

Summarizing, once chosen the numerical values (Δ_E, T_E) and ($\Delta_F = 180°$, T_F) we can establish the following system of equations:

1.- The formulas (7.6) and (7.6') applied to any point P in the zone EF (and in particular to the point E), are

$$\Delta_P^{(m)} = \frac{\mu_3}{p_0} + \frac{\pi}{2} - I_c + \frac{\cos I_3 - \cos I_3'}{A_3\, p_0} \qquad (15.1)$$

$$T_P^{(m)} = \sum_{i=0}^{3} \frac{1}{A_i} \log \frac{\tan (I_i'/2)}{\tan (I_i/2)} + \frac{\cos I_c}{k_c} \tag{15.2}$$

therefore, if we make

$$H = \frac{\pi}{2} - \Delta_P^{(m)} + \frac{1}{p_0} \left[\mu_3 + \frac{\cos I_3}{A_3} \right] \tag{15.3}$$

$$K = T_P^{(m)} - \sum_{i=0}^{2} \frac{1}{A_i} \log \frac{\tan (I_i'/2)}{\tan (I_i/2)} + \frac{1}{A_3} \log \tan \frac{I_3}{2} \tag{15.4}$$

the equations (15.1) and (15.2) adopt the form

$$H = \frac{\cos I_3'}{A_3\, p_0} + I_c \tag{15.5}$$

$$K = \frac{1}{A_3} \log \tan \frac{I_3'}{2} + \frac{\cos I_c}{k_c} \tag{15.6}$$

To these equations we have to add the conditions determined by the laws of refraction, i.e.

$$p_0 = \frac{\sin I_0}{k_0} = \frac{\sin I_3'}{k_3'} \tag{15.7}$$

$$p_0 = \frac{\sin I_0}{k_0} = \frac{\sin I_c}{k_c} \tag{15.8}$$

In a similar way, the equation (15.2) applied to the axial ray, for which the equalities $I_0 = I_0' = I_1 = I_1' = I_2 = I_2' = I_3 = I_3' = I_c = 0$ are verified, enable us to write

$$T_F^{(m)} = \sum_{i=0}^{3} \frac{1}{A_i} \log \frac{k_i'}{k_i} + \frac{1}{k_c}$$

and therefore, if we make

$$L = T_F^{(m)} - \sum_{i=0}^{2} \frac{1}{A_i} \log \frac{k_i'}{k_i} + \frac{1}{A_3} \log k_3 \tag{15.9}$$

we shall obtain the equation

$$L = \frac{1}{A_3} \log k_3' + \frac{1}{k_c} \tag{15.10}$$

The system formed by the five equations (15.5), (15.6), (15.7), (15.8), and (15.10), contains the unknown quantities l_0, l_3', l_c, k_3', k_c, and can be resolved as we shall see.

In fact, from (15.5) is deduced

$$\cos l_3' = A_3\, p_0\, (H - l_c) \tag{15.11}$$

and from (15.10)

$$\log \sin l_3' = A_3 \left(L - \frac{p_0}{\sin l_c} \right) + \log p_0$$

Elimination of l_3' between both equations gives us the fundamental equality

$$2A_3 \left(L - \frac{p_0}{\sin l_c} \right) = \log \frac{1 - A_3^2\, p_0^2\, (H - l_c)^2}{p_0^2} \tag{15.12}$$

in which H and p_0 depend on l_0, while $L = -169.809540578$ is now a constant.

With the purpose of obtaining another equality that only depends on l_0 and l_c, we should furthermore consider the equation (15.6), which has not intervened in the obtention of (15.12).

So, from (15.11) is deduced

$$\tan \frac{l_3'}{2} = \frac{1 - A_3\, p_0\, (H - l_c)}{1 + A_3\, p_0\, (H - l_c)}$$

from where, substituting in (15.6), comes the equation

$$2A_3 \left(K - \frac{p_0}{\tan l_c} \right) = \log \frac{1 - A_3\, p_0\, (H - l_c)}{1 + A_3\, p_0\, (H - l_c)} \tag{15.13}$$

in which H, K and p_0 depend on the value assigned to l_0.

The resolution of the system formed by the equations (15.12) and (15.13), with respect to the unknown quantities I_0, I_c, must lead, through a numerical treatment, to the solution of the problem proposed.

With this object, different calculations have been carried out which are summarized below:

1.- Starting from the data $\Delta_E = 146°$, $\Delta_F = 180°$, $T_F = 20^m 11^s$, taken from the Bull. (1968) and from the constants A_0, A_1, A_2, A_3, k_0, k_1, k_2, k_3, already obtained, for values given for I_0, we have calculated the corresponding values of I_c by means of the equation (15.12). Applying these quantities to the equation (15.13) we have obtained the corresponding values of K, and finally those of $T_E^{(m)}$ through the equality (15.4)

$$T_E^{(m)} = K + \sum_{i=0}^{2} \frac{1}{A_i} \log \frac{\tan (I_i'/2)}{\tan (I_i/2)} - \frac{1}{A_3} \log \tan \frac{I_3}{2}$$

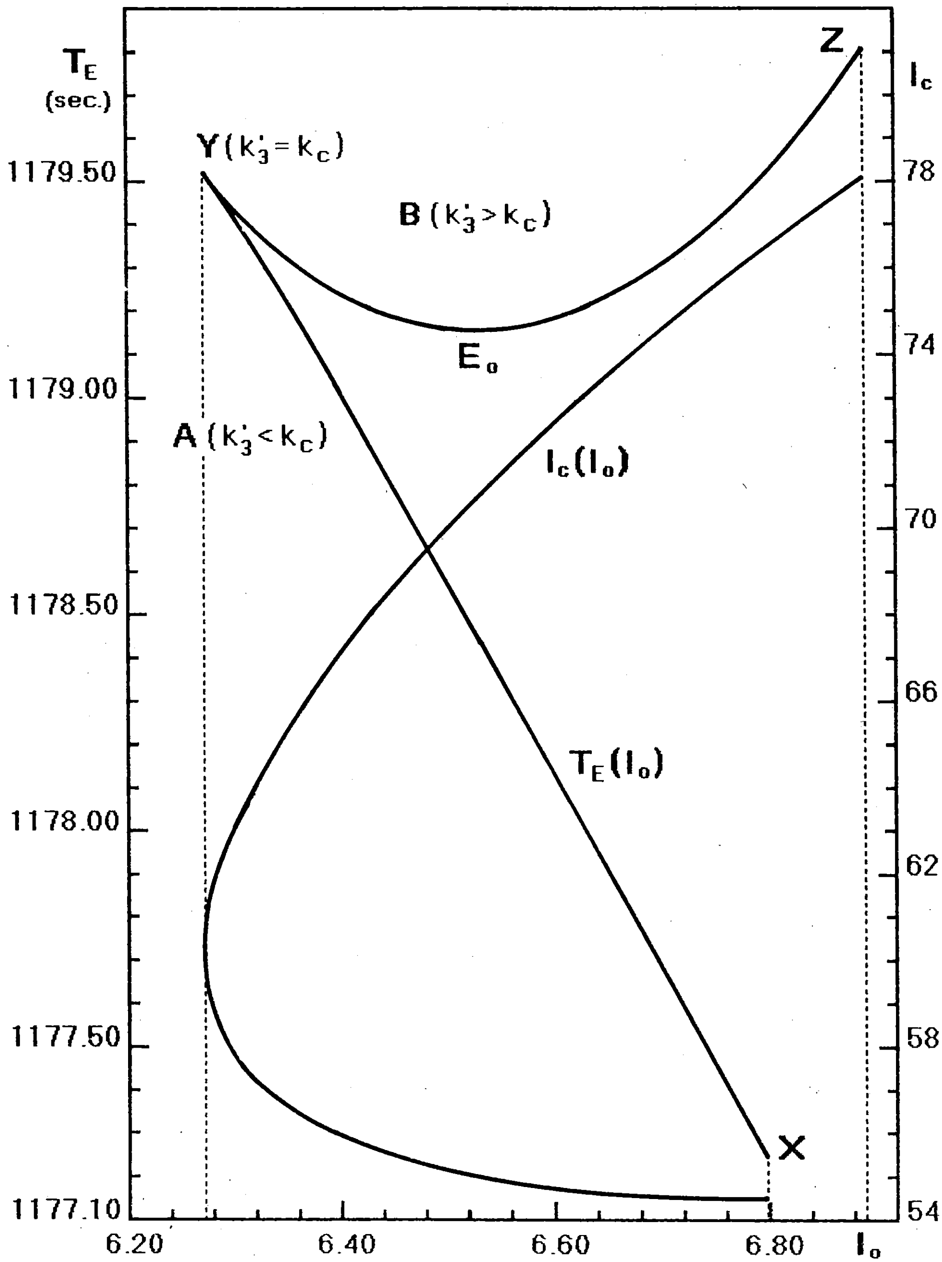
The results obtained have been summarized in figure 15.1 which defines the correspondence between the values (I_0, I_c), (I_0, T_E).

In an analogous way figure 15.2 which defines the correspondence of values for I_0 with those of k_3', k_c, and R_c, has been constructed.

All of these results, including moreover the corresponding values of v_3', and v_c, are contained in table XII_1.

We should observe that the curve which defines the correspondence (I_0, T_E) in figure 15.1, presents two branches which determine two quite different regions, which we denominate regions A and B.

In the first, the solutions of the system define an almost straight line which extends from the point Y (I_0 = 6°.2721, T_E = 1179^s.5183) to the point X (I_0 = 6°.7965, T_E = 1177^s.2464) and over which $k_3' \leq k_c$, verifying the equality $k_3' = k_c$ at point Y.



T_E
(sec.)
1179.50
1179.00
1178.50
1178.00
1177.50
1177.10
6.20
6.40
6.60
6.80
I_o
I_c
78
74
70
66
62
58
54
Z
Y ($k_3' = k_c$)
B ($k_3' > k_c$)
E_o
A ($k_3' < k_c$)
$I_c(I_o)$
$T_E(I_o)$
X

Fig. 15.1

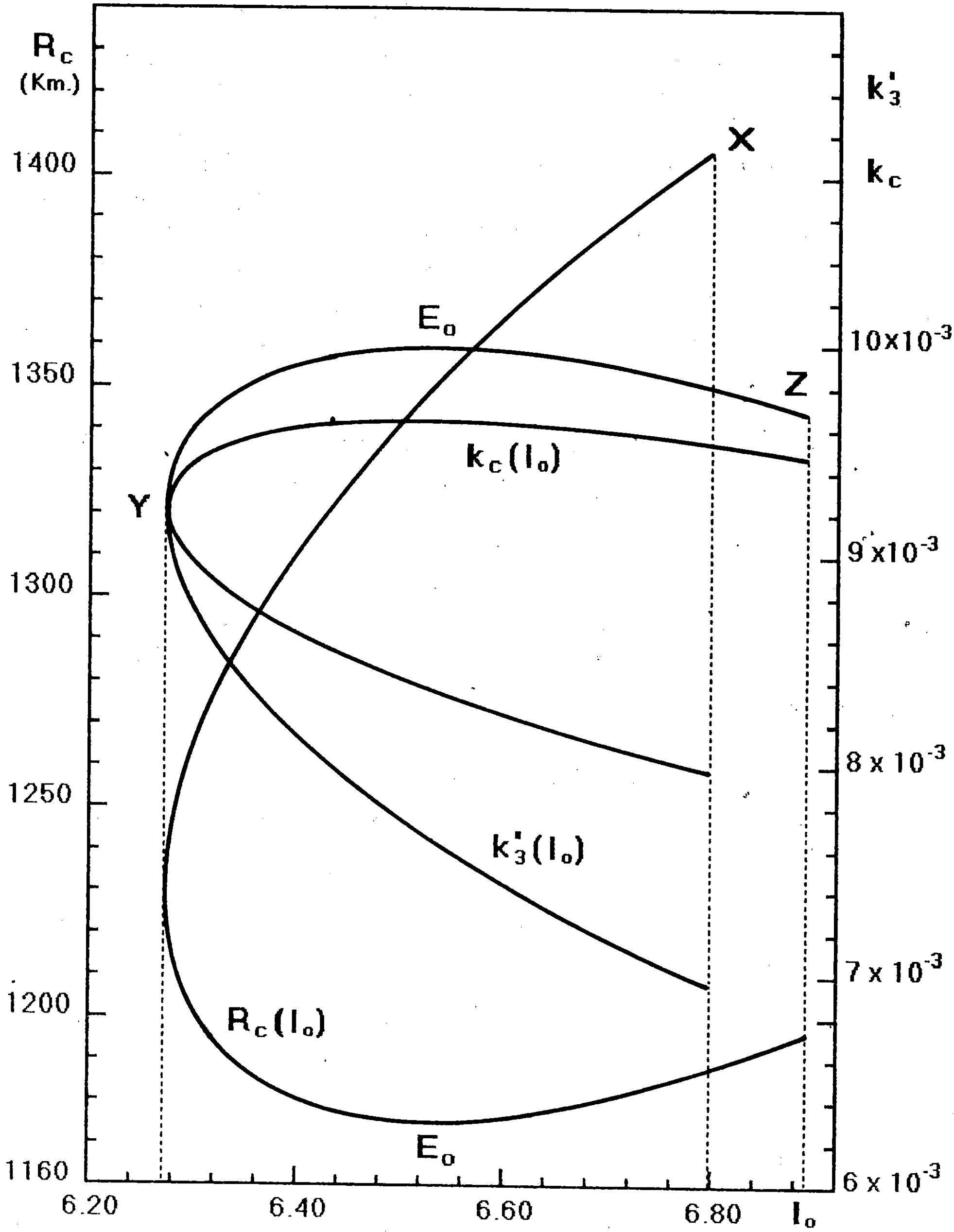

Rc
(Km.)
1400
1350
1300
1250
1200
1160
k'3
kc
10x10-3
9x10-3
8 x 10-3
7 x 10-3
6 x 10-3
X
Y
Z
E0
kc(I0)
k'3(I0)
Rc(I0)
E0
6.20
6.40
6.60
6.80
I0

Fig. 15.2

In region B, the corresponding curve, similar to a catenary, extends from point Y to point $Z\,(I_0 = 6^\circ.8903,\ T_E = 1179^s.8091)$. presenting a minimum at point $E_o\,(I_0 = 6^\circ.5281,\ T_E = 1179^s.1569)$ whose significance we shall see, further on. In this region the condition $k_3' \geq k_c$ is verified.

The definition of points X and Z, as extremes of the corresponding curves, is determined in the case of point X by the fact that for values higher than $I_0 = 6^\circ.7965$ the equation (15.10) is not satisfied and therefore equation (15.12) is not verified. In the case of point Z, the solutions with values higher than $I_0 = 6^\circ.8903$ lead to results with $I_3' > 90^\circ$, which do not make sense.

In order to determine the best solution to the problem proposed, we have to take into account the following considerations:

1.- Given the time assigned to the point $\Delta = 146^\circ$ in the tables of the Bull. (1968), which is $T = 1180^s.5$, it seems natural that the time T_E of the aforementioned curves should approximate the most possible to the said value.

2.- The definition of the point D-cusp. ($\Delta = 122^\circ$) given by Choy and Cormier, demands to a certain extend its existence in the solution adopted.

3.- Whatever the solution adopted, it must lead to values (Δ, T) concordant with the Tables of Bull. (1968).

Let us see, then, the results which follow with the most significant solutions:

First case.

Let us suppose that the solution contains the point $E_o\,(\Delta = 146^\circ,\ T = 1179^s.15694997)$.

Evidently, in this case, the solution satisfies the equation (15.12) and (15.13), with the following results:

$I_0 = 6°.52805877193$

$I_1 = 11°.3148644414$

$I_2 = 12°.1588144891$

$I_3 = 40°.9728597311$

$I_c = 70°.6839045575$

$H = 1.36151628422$

$I'_0 = 11°.3148644414$

$I'_1 = 22°.2363683121$

$I'_2 = 40°.9728597311$

$I'_3 = 77°.8294994108$

$K = 21.6127890400$

$k'_3 = 9.98270271059 \times 10^{-3}$

$k_c = 9.63736092798 \times 10^{-3}$

$R'_3 = R_c = 1174.91960427$ km.

$z'_3 = z_c = 5196.1084$ km.

$v'_3 = 11.7288731183$ km./sec.

$v_c = 11.3231242877$ km./sec.

With the values which the aforementioned solution provides us with, it is also proved that the equality (12.20) is satisfied, that is

$$\tan I_c = \frac{\nu_4}{p_0} = \frac{\nu_c}{p_0}$$

or what is the same $\Phi_c = 0$, agreeing with (12.17).

As a consequence of this we can say that the epicentral distance $\Delta = 146°$, belonging to the waves PKIKP, is a minimum, given that $\Psi_c = 3948.0038 > 0$. The same result would have been obtained by numerical calculation in resolving the system formed by the equations (15.12) and (15.14)

$$\tan I_c = M + \frac{1}{A_3^2\, p_0^2\, (H - I_c)} \qquad (15.14)$$

where

$$M = \frac{1}{p_0}\left(\nu_3 - \frac{1}{A_3 \cos I_3}\right)$$

In table $XIII_1$ the values of Δ and T figure for PKiKP and PKIKP waves.

Figures $(15.3)_1$, $(15.4)_1$ and $(15.5)_1$ reflect the behaviour of the functions $\Delta(I_0)$, v(r), and $T(\Delta)$, in the intervals ($I_0 = 10°$, $I_0 = 0°$), ($r = R_3$, $r = 0$), and ($\Delta = 100°$, $\Delta = 180°$) respectively.

From these results the following conclusions are deduced:

a) The D-cusp. ($\Delta = 122°$) of Choy-Cormier, which we have taken as minimum of the waves $PKP^{(T)}$ appears essentially as an extreme of these waves. with $I'_3 = 90°$, determining the beginning of the inner core.

In fact. if we calculate the angle I_0 through the formula

$$I_0 = \text{arc sin} \frac{k_0}{k'_3} = 6°.67882889478$$

we shall have exactly defined the beginning of the inner core, and to the said angle I_0 correspond the values

$\Delta = 122°.008732007$ $\quad\quad$ $T = 1135^s.91521975$

which reflect very closely the previous affirmation.

This ray tangent to the sphere of radius R_c, since $k'_3 > k_c$, is refracted in the inner core. obtaining

$\Delta = 152°.237781030$ $\quad\quad$ $T = 1190^s.02738803$

which we could interpret as the second cusp $C = 154° \pm 2°$ of Choy-Cormier, although they find it to be the epicentral distance of the last PKP ray tangent to the sphere of radius R_c.

As the two interpretations are different. in what follows we shall refer to D'-cusp. to designate the last ray of the waves $PKP^{(T)}$, and to D"-cusp. for the first refracted ray of the waves PKIKP.

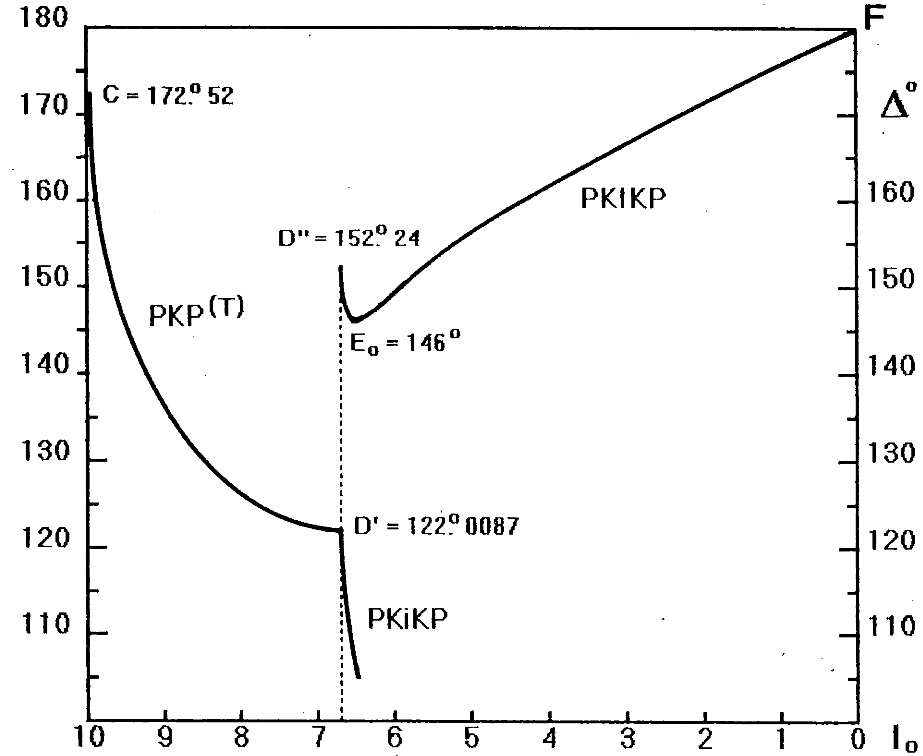


Fig. $(15.3)_1$

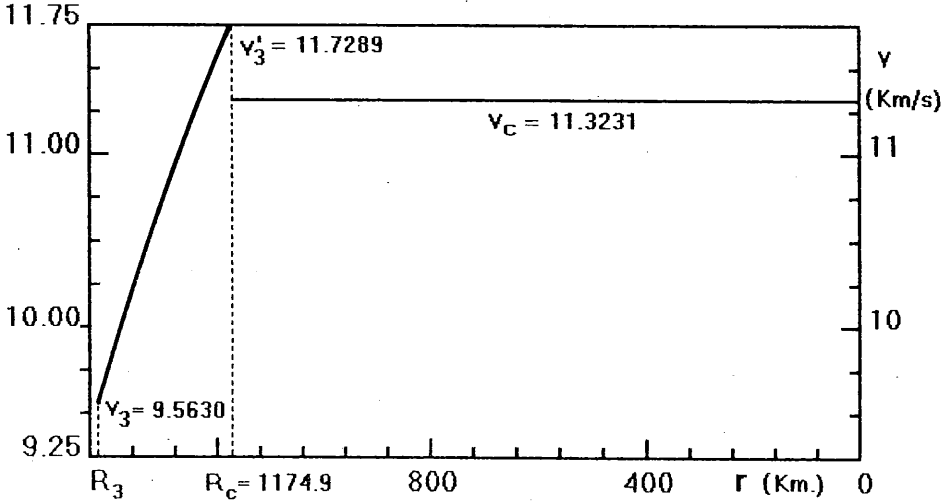


Fig. $(15.4)_1$

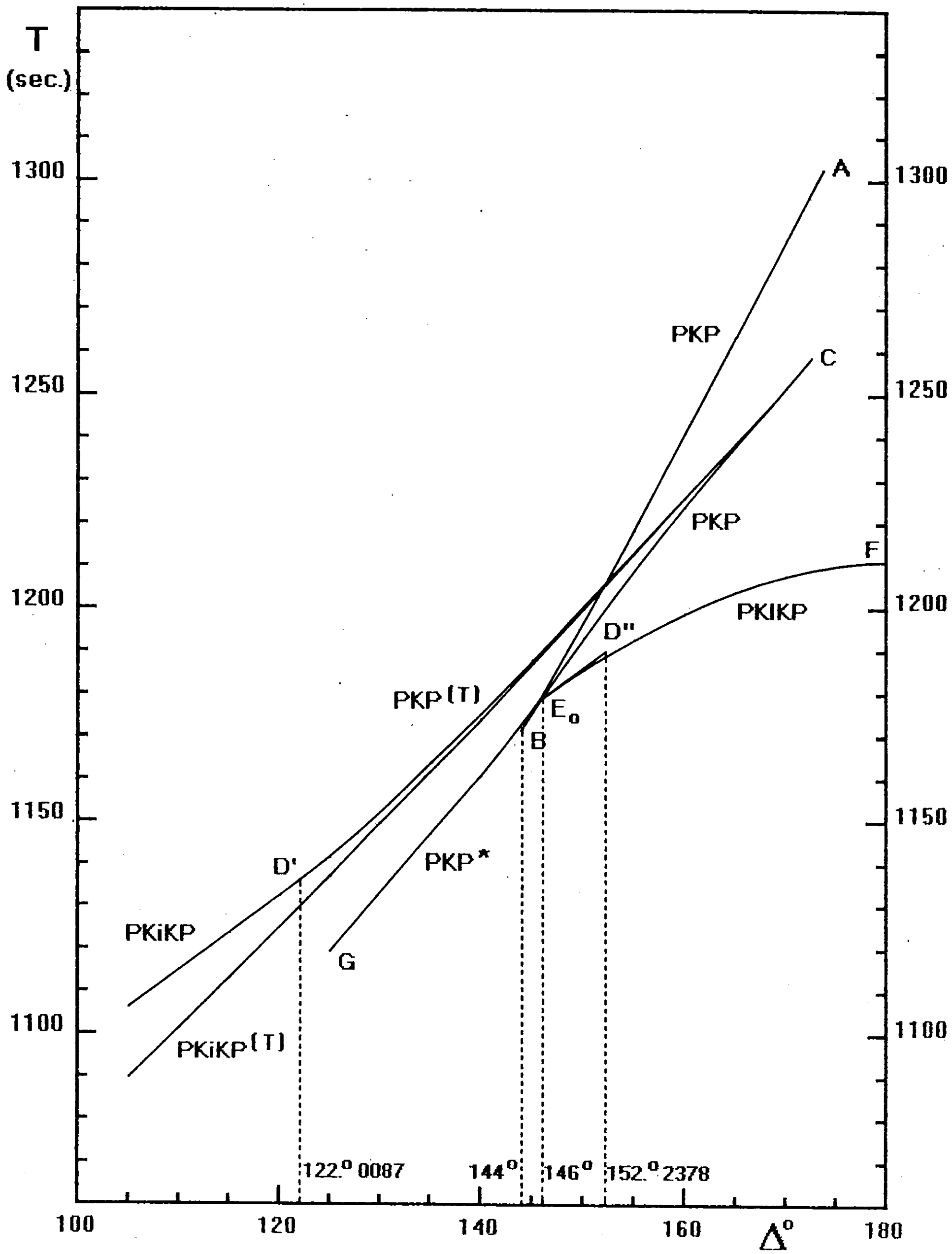

T
(sec.)
1300
1250
1200
1150
1100
A
PKP
C
PKP
F
PKIKP
D"
PKP (T)
E o
B
PKP *
D'
PKiKP
G
PKiKP (T)
122.° 0087
144°
146°
152.° 2378
100
120
140
160
Δ°
180

Fig. (15.5)₁

b) The concentration of arrival energy that exists in the minimum E_0 (Δ = 146°) agrees with the fact that it figures as the first observable value not extrapolated from the Bull. (1968) tables, although other authors (Jeffreys) find Δ = 145°.

c) We should observe that the travel-times of the decreasing branch $D''E_0$ become confused with those of the increasing branch E_0F, which could make their identification difficult.

d) Finally, let us say that the zone from 110° to 122° could be generated by PKiKP waves and the zone 122° - 139° by $PKP^{(T)}$ and $PKiKP^{(T)}$ waves, understanding for these last waves those which are reflected at R_3 and which figure in table XI.

In agreement with table XII_1, table XII_2 has been constructed, and for every R_c this table contains the values (I_0, Δ and T) of the D'-cusp. point, which is to say that from the point for which $I_3' = 90°$, while the values (I_0, Δ and T) for the D"-cusp. point are determined by the condition $I_c = 90°$ in region A and by the condition $I_3' = 90°$ in region B.

Second case.

Imposing the condition

$$R_c = 1216 \text{ km.} = R_3 \exp \frac{k_3 - k_3'}{A_3}$$

the value $k_3' = 9.40396865058 \times 10^{-3}$ is easily obtained and from the equation (15.10) is deduced

$$k_c = \frac{A_3}{A_3 L - \log k_3'} = 9.31885404500 \times 10^{-3}$$

Also, from the curves of coordinates (I_0, k_3') or (I_0, R_c) the approximate value of I_u is determined, which can also be found by numerical calculation.

The resolution of the system of equations (15.12) and (15.13) leads to the following results:

$I_0 = 6°.27764362313$ $\qquad$ $I'_0 = 10°.8772232055$

$I_1 = 10°.8772232055$ $\qquad$ $I'_1 = 21°.3442860014$

$I_2 = 11°.6876213157$ $\qquad$ $I'_2 = 39°.0982143137$

$I_3 = 39°.0982143137$ $\qquad$ $I'_3 = 62°.3346253306$

$I_c = 61°.3611520417$

$H = 1.36370997381$ $\qquad$ $K = 21.5748333640$

$\Delta = 146°$ $\qquad$ $T = 1179^s.50082577$

$$k'_3 = 9.40396865058 \times 10^{-3}$$

$$k_c = 9.31885404500 \times 10^{-3}$$

$$z'_3 = z_c = 5155.028 \text{ km.}$$

$$v'_3 = 11.4352258791 \text{ km./sec.}$$

$$v_c = 11.3317265187 \text{ km./sec.}$$

In this case the travel-times obtained are somewhat better, with errors $|T - T_t|$ smaller than 1 sec. with respect to the Bull. (1968) tables. However, we must emphasize the following:

a) The last ray of the $PKP^{(T)}$ waves is found for the angle

$$I_0 = \text{arc sin} \frac{k_0}{k'_3} = = 7°.09190363064$$

with an epicentral distance

$$\Delta = 122°.508819747 \quad \text{(D'-cusp.)}$$

for which reason the point D-cusp. of Choy-Cormier is not obtainable.

b) The PKIKP waves initiate their trajectory at D"-cusp. with an epicentral distance $\Delta = 137°.937774234$, having a minimum for $\Delta = 135°.2090$ and an initial angle $I_0 = 7°.0595$.

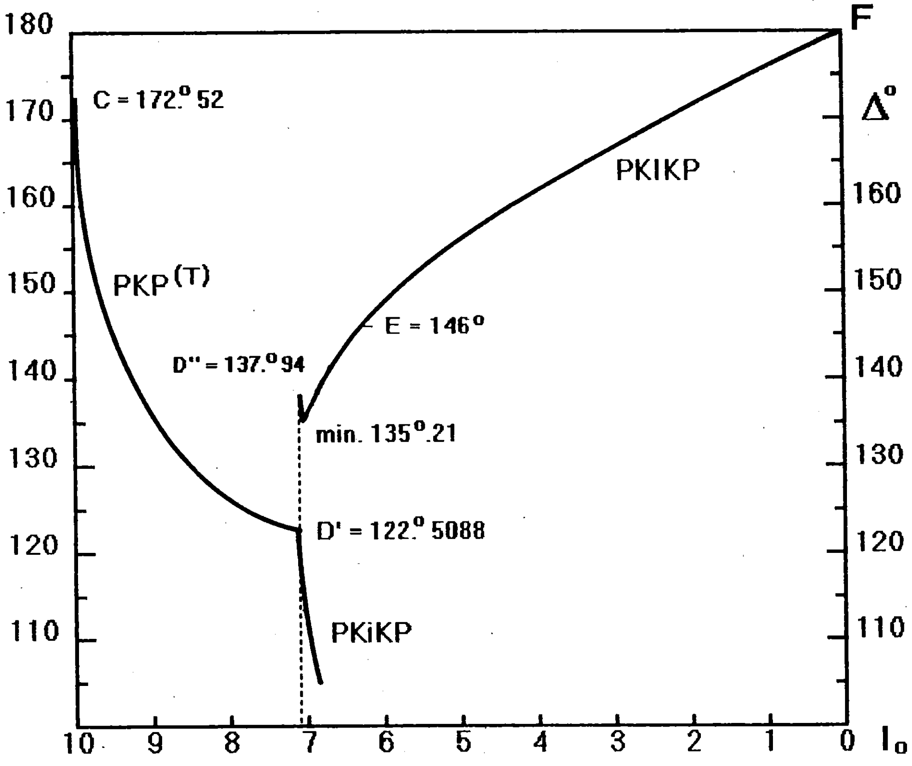


Fig. $(15.3)_2$

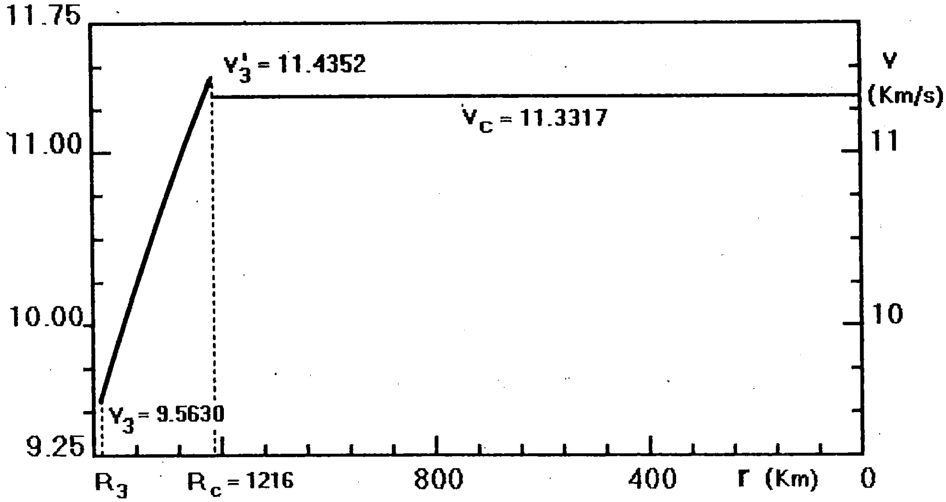


Fig. $(15.4)_2$

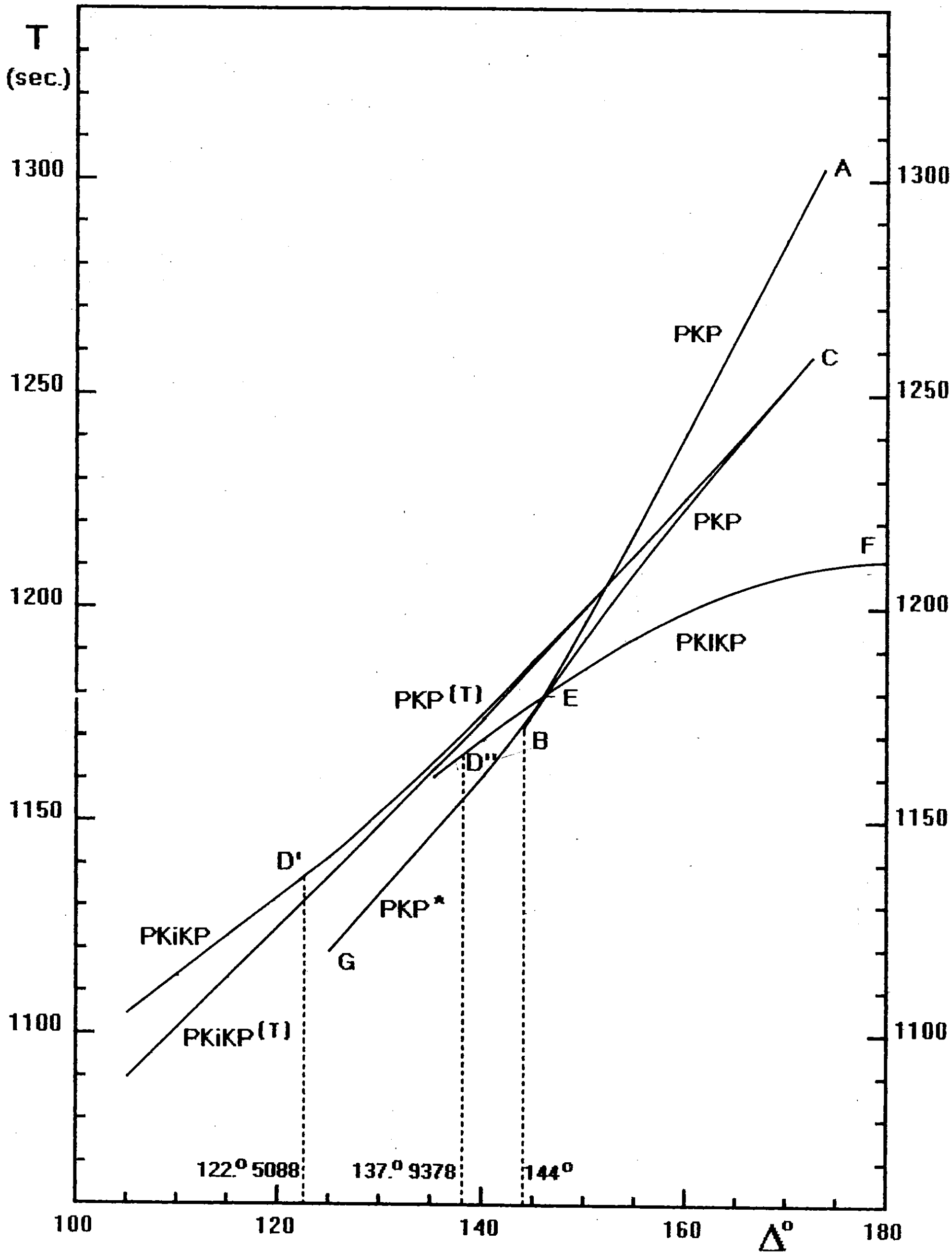

T
(sec.)
1300
1250
1200
1150
1100
A
C
F
PKP
PKP
PKIKP
PKP (T)
E
B
D''
D'
PKiKP
PKP*
G
PKiKP (T)
122.° 5088
137.° 9378
144°
100
120
140
160
180
Δ°

Fig. $(15.5)_2$

c) The difference between v_3' and v_c becomes very small, as the velocity only decreases 0.1 km./sec. on passing to the inner core, while in the previous case it was 0.4 km./sec.

Table $XIII_2$, as well as the graphs of the functions $\Delta(I_0)$, $v = v(r)$ and $T(\Delta)$ of figures $(15.3)_2$, $(15,4)_2$ and $(15.5)_2$, clearly reflect the behaviour of this solution.

As in the previous case we can observe that the travel-times T of the decreasing branch of the PKIKP waves become confused with those of the increasing branch, so the first observable epicentral distance should be that of the minimum $\Delta = 135°.2$. However, its identification seems to be hindered by the fact that the $PKP^{(T)}$ and $PKiKP^{(T)}$ waves can lead us to other interpretations because there is travel-time overlapping.

Third case.

This solution is determined by the condition $k_3' = k_c$, which corresponds in the graph (I_0, T_E) to the point Y.

In this case the equation (15.10), i.e.

$$L = \frac{1}{A_3} \log k_3' + \frac{1}{k_c}$$

enables us to easily obtain

$$k_3' = k_c = 9.21574213746 \times 10^{-3}$$
$$R_3' = R_c = 1229.66806810 \text{ km.}$$
$$z_3' = z_c = 5141.3599 \text{ km.}$$
$$v_3' = v_c = 11.3323038303 \text{ km./sec.}$$

and therefore

$I_0 = 6°.27210450452$ $\quad$ $I_0' = 10°.8675476412$

$I_1 = 10°.8675476412$ $\quad$ $I_1' = 21°.3246106182$

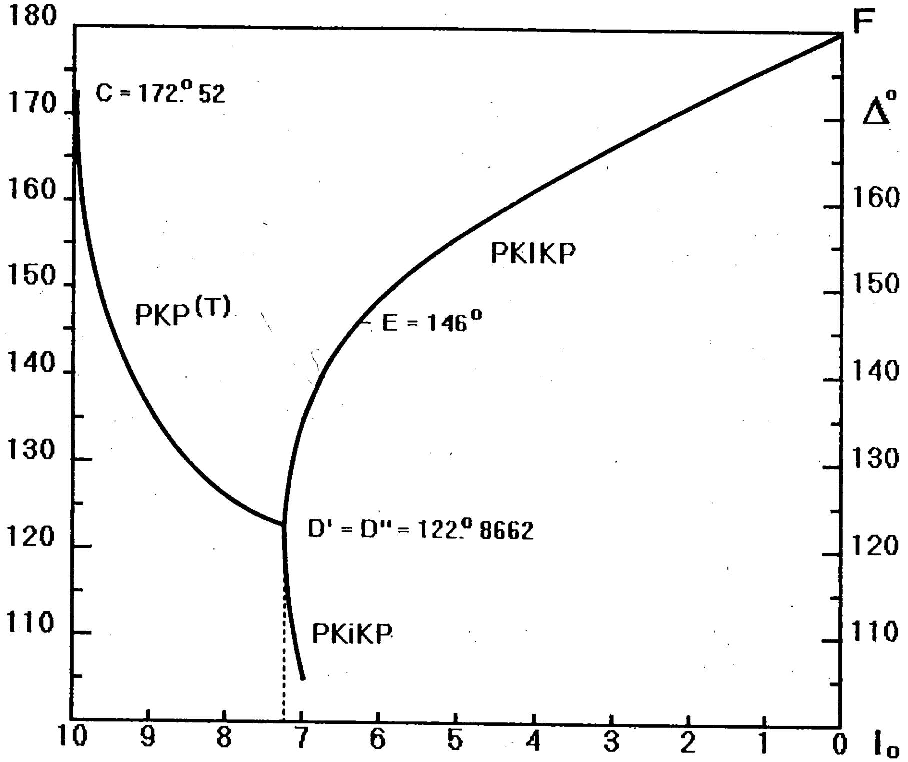


Fig. $(15.3)_3$

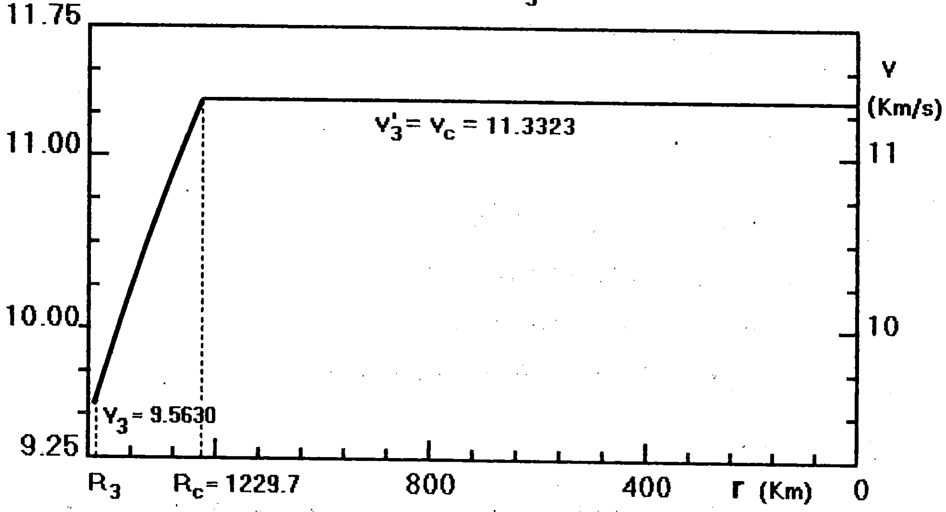
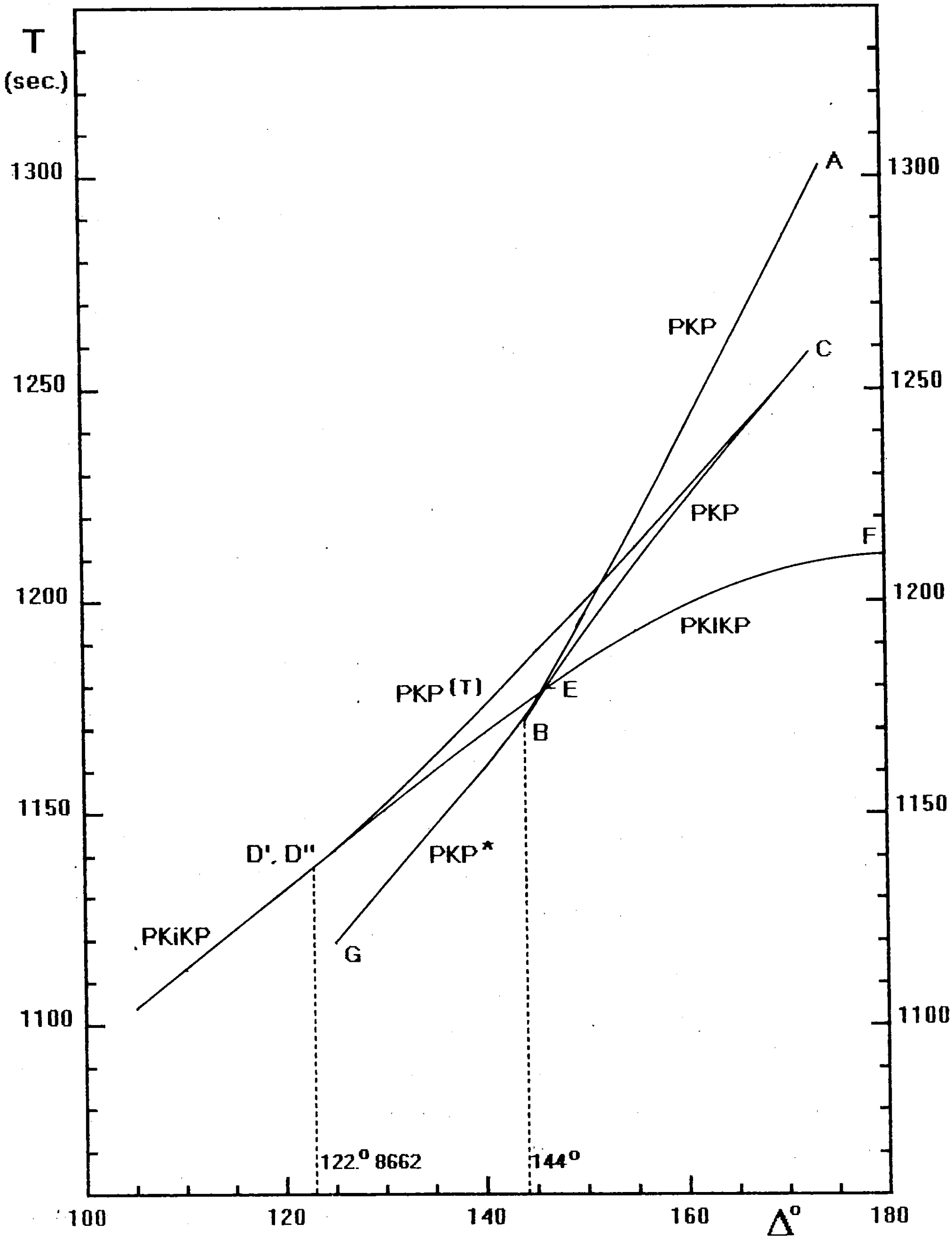


Fig. $(15.4)_3$



T
(sec.)
1300
1250
1200
1150
1100
1300
1250
1200
1150
1100
A
PKP
C
PKP
F
PKIKP
PKP (T)
E
B
D', D''
PKP*
PKiKP
G
122.° 8662
144°
100
120
140
160
180
Δ°

Fig. $(15.5)_3$

$I_2 = 11°.6772052305$ $\qquad$ $I'_2 = 39°.0573079277$

$I_3 = 39°.0573079277$ $\qquad$ $I'_3 = 60°.1330627879$

$I_c = 60°.1330627879$

$H = 1.36378827569$ $\qquad$ $K = 21.5762055690$

$\Delta = 146°$ $\qquad$ $T = 1179^s.51831276$

In the same way it has to be emphasized that this solution is the optimum one as regards travel-times T, as is found to be true in table $XIII_3$.

As before, the point D-cusp. ($\Delta = 122°$) of Choy-Cormier is not obtainable, since the end of the $PKP^{(T)}$ waves and the beginning of the PKIKP waves is found for an epicentral distance $\Delta = 122°.866211069$, with an initial angle

$$I_0 = \arc\sin \frac{k_0}{k'_3} = 7°.23751936747$$

In this way the D'-cusp. and D"-cusp. rays become confused. The radius R_c obtained is a little greater than the one obtained in the preceding case and the angle I_0 corresponding to this case is a minimum, as table XII_1 proves.

The graphs of the functions $\Delta(I_0)$, $v(r)$ and $T(\Delta)$ of figures $(15.3)_3$, $(15.4)_3$ and $(15.5)_3$ reflect these characteristics and the fact that the PKIKP waves do not have a minimum, but rather that the branch D"F is constantly increasing.

Fourth case.

The solution to the system of equations (15.12) and (15.13) is chosen in such a way that the beginning of the PKIKP waves corresponds to the epicentral distance $\Delta = 110°$.

This solution, which exists for a point which is very close to point Y of the region A ($k'_3 < k_c$), adapts itself in part to interpretations of Jeffreys, Gutenberg and Miss Lehmann.

The results obtained in this case are the following:

$I_0 = 6^\circ.30229211309$

$I_1 = 10^\circ.9202810023$

$I_2 = 11^\circ.7339752005$

$I_3 = 39^\circ.2805269122$

$I_c = 57^\circ.7432322800$

$H = 1.36337742544$

$\Delta = 146^\circ$

$I'_0 = 10^\circ.9202810023$

$I'_1 = 21^\circ.4318688943$

$I'_2 = 39^\circ.2805269122$

$I'_3 = 55^\circ.5191141340$

$K = 21.5589695420$

$T = 1179^s.40633781$

$$k'_3 = 8.71837040052 \times 10^{-3}$$

$$k_c = 8.94417541412 \times 10^{-3}$$

$$R'_3 = R_c = 1266.52860689 \text{ km.}$$

$$z'_3 = z_c = 5104.4994 \text{ km.}$$

$$v'_3 = 11.0420655177 \text{ km./sec.}$$

$$v_c = 11.3280540270 \text{ km./sec.}$$

We should make note of the fact that the depth $z_c = 5104.4994$ km. which defines the beginning of the inner core quite closely approximates to the depth $z_c = 5120$ km. accepted by the aforementioned authors.

If we focus our attention on the graphs of the functions $\Delta(I_0)$ and $v(r)$, figures $(15.3)_4$ and $(15.4)_4$, we observe that the point D'-cusp. is found to have an epicentral distance $\Delta = 124^\circ.409996684$ for an initial angle $I_0 = 7^\circ.65281543509$ while for the point D"-cusp., one has $\Delta = 110^\circ$, and $I_0 = 7^\circ.45849886997$. Figure $(15.5)_4$ reflect the behaviour of the function $T(\Delta)$.

In table $XIII_4$, which summarize the results in this case, it is proved that the differences $|T - T_t|$ are inferior to 1.1 sec.

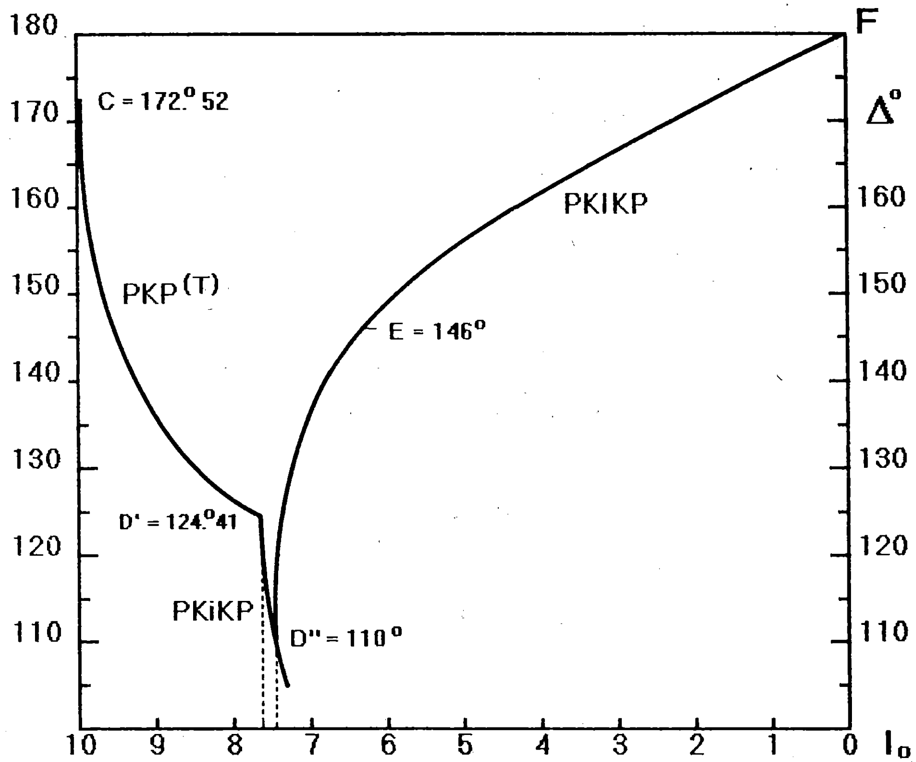


Fig. (15.3)$_4$

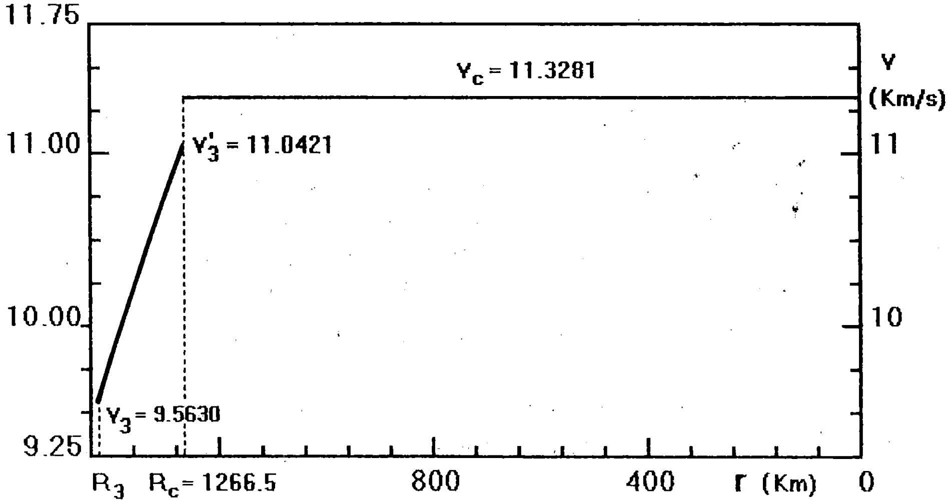


Fig. (15.4)$_4$

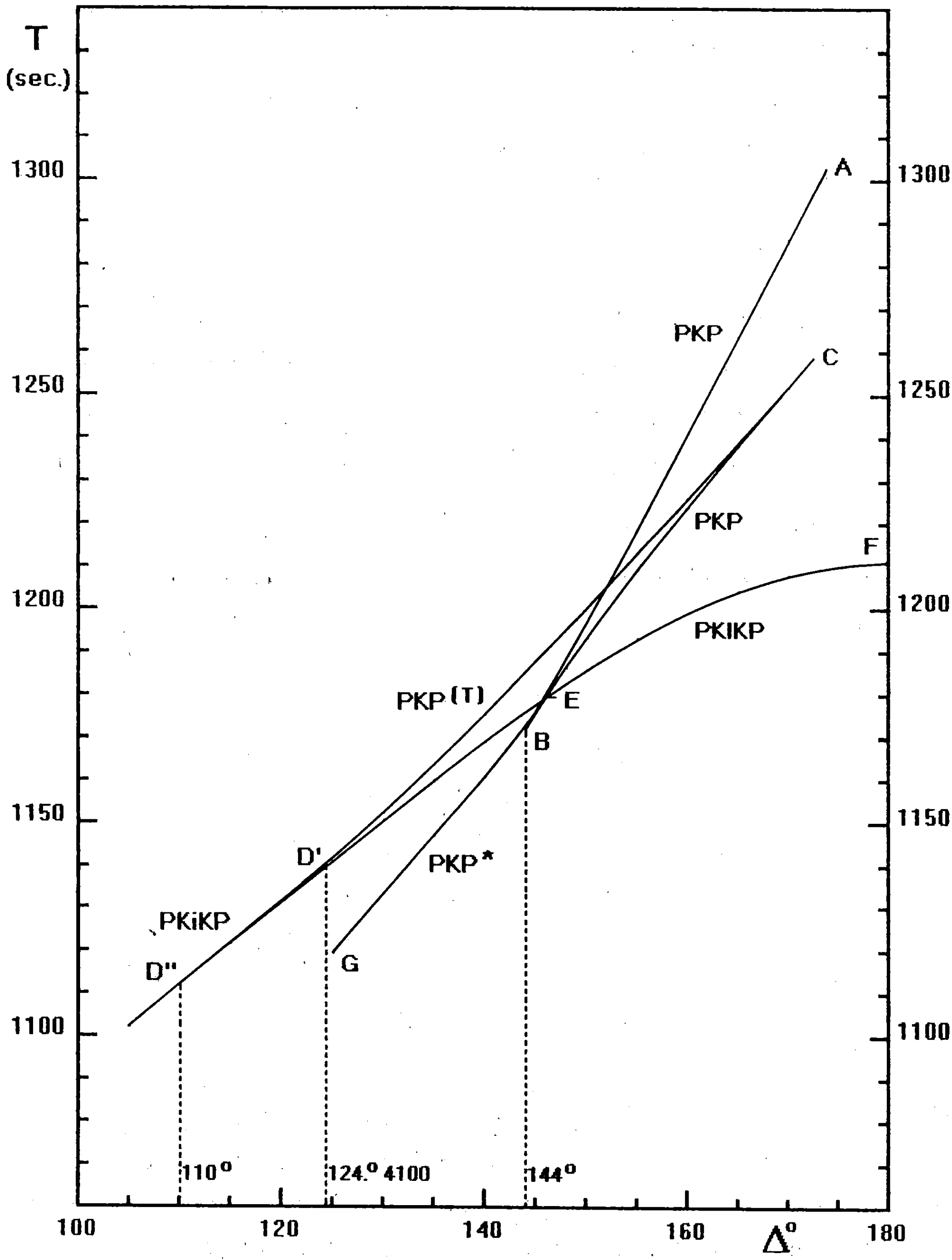

T
(sec.)
1300
1250
1200
1150
1100
1300
1250
1200
1150
1100
A
C
F
PKP
PKP
PKIKP
PKP (T)
E
B
D'
PKP*
PKiKP
D''
G
110°
124.° 4100
144°
100
120
140
160
180
Δ°

Fig. $(15.5)_4$

Fifth case.

It is assumed, in this case, that the solution to the system of equations (15.12) and (15.13), contains the point $Z\,(I'_3 = 90°)$, extreme of the region $B\,(k'_3 < k_c)$.

The said resolution is simplified by the evident equalities $H = I_c$ and $K \tan I_c = p_0$.

The corresponding results for this case are the following:

$I_0 = 6°.89028853157$	$I'_0 = 11°.9487222865$
$I_1 = 11°.9487222865$	$I'_1 = 23°.5361780076$
$I_2 = 12°.8414722992$	$I'_2 = 43°.7818249857$
$I_3 = 43°.7818249857$	$I'_3 = 90°$
$I_c = 78°.0847309617$	
$H = 1.36283565082$	$K = 21.8037675260$
$\Delta = 146°$	$T = 1179^s.80911952$

$$k'_3 = 9.67774938713 \times 10^{-3}$$
$$k_c = 9.46923257922 \times 10^{-3}$$
$$R'_3 = R_c = 1196.39015261 \text{ km.}$$
$$z'_3 = z_c = 5174.6378 \text{ km.}$$
$$v'_3 = 11.5783640662 \text{ km./sec.}$$
$$v_c = 11.3288966106 \text{ km./sec.}$$

In this solution it is necessary to signify some of its characteristics:

a) The time corresponding to point Z, which coincides with the point D"-cusp., is in its turn the closest to T_E observed, since the difference $|T - T_t| = 0{,}691$ sec. is the smallest when it is compared with that obtained in the previous cases.

b) The point D'-cusp. is found for an epicentral distance of 122°.169461924

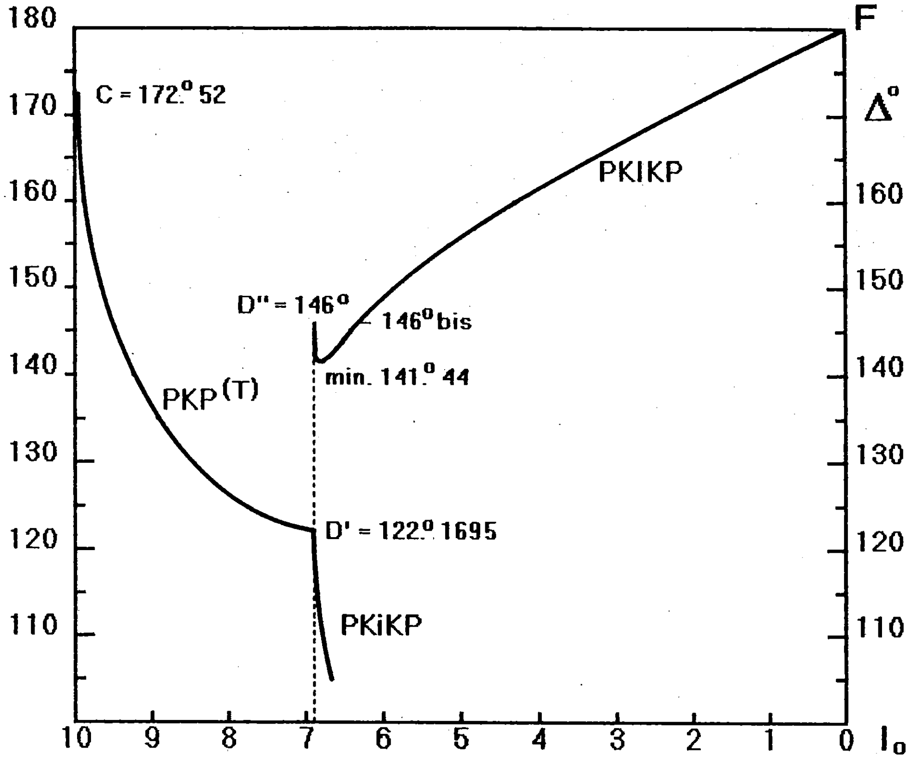


Fig. $(15.3)_5$

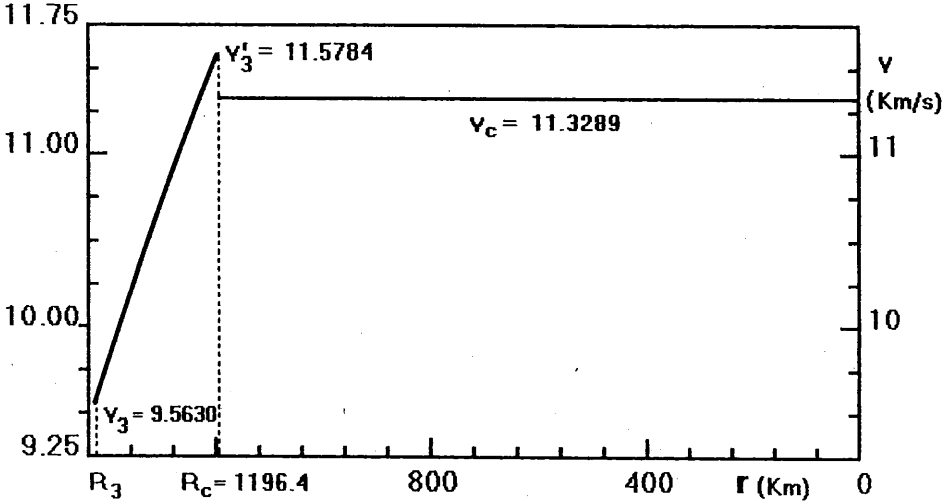
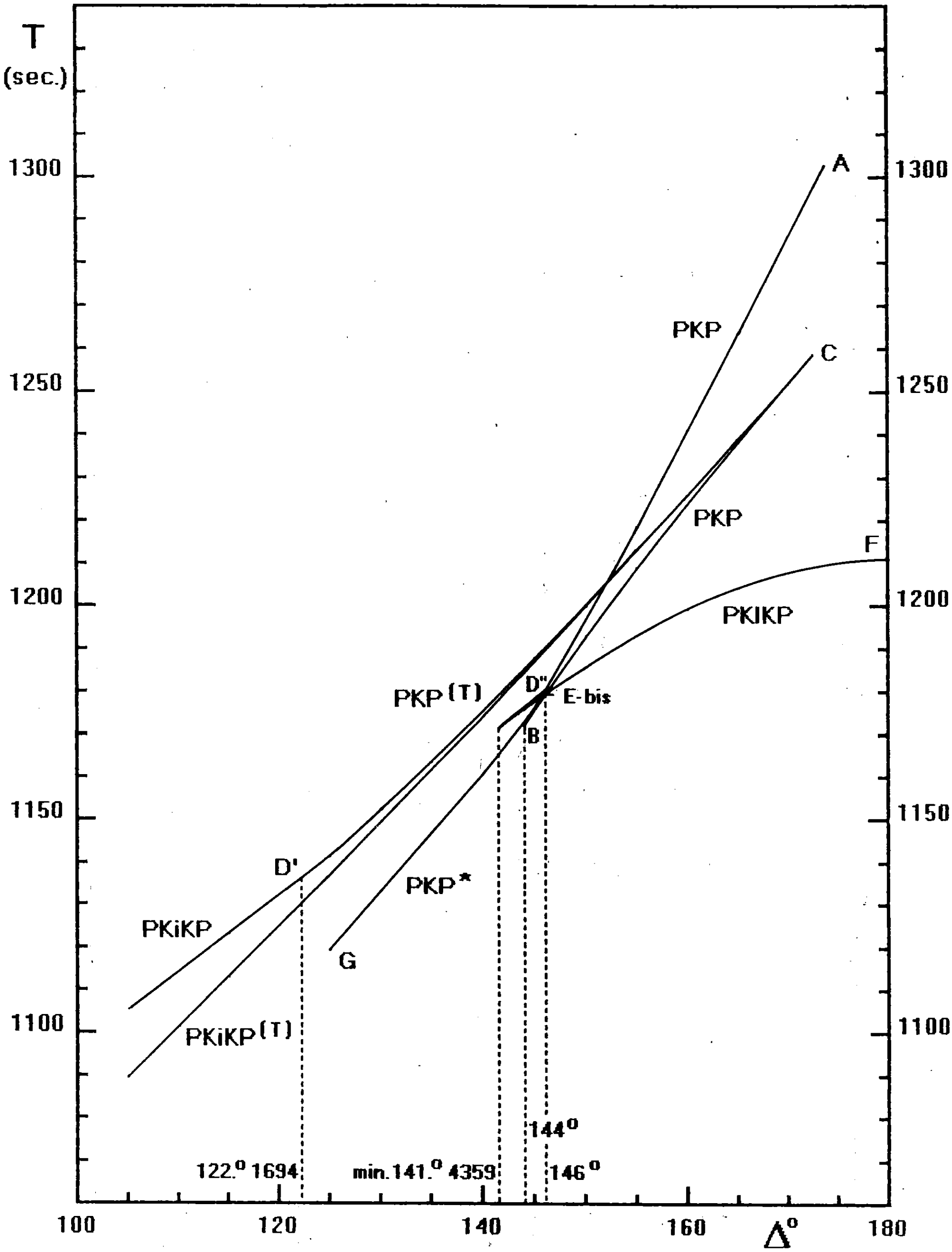


Fig. $(15.4)_5$



T
(sec.)
1300
1250
1200
1150
1100
A
C
F
PKP
PKP
PKIKP
PKP (T)
D"
E-bis
B
PKP*
D'
PKiKP
G
PKiKP (T)
144°
122.° 1694
min.141.° 4359
146°
100
120
140
160
180
Δ°

Fig. $(15.5)_5$

c) The initial angle I_0 = 6°.89028853157 for point Z and points D'-cusp. and D"-cusp. is the same.

d) The PKIKP waves initiate their trajectory at D"-cusp. with the epicentral distance Δ = 146°.

e) The PKIKP waves present a minimum for Δ = 141°.4359 with an initial angle I_0 = 6°.8043.

f) In the increasing branch, for I_0 = 6°.31281736167 we found the second point Δ = 146° which is the solution that also figures in tables XII_1 and XII_2 in region B. These solutions are the same.

The graphs of the functions $\Delta(I_0)$, v = v(r) and $T(\Delta)$ of figures $(15.3)_5$, $(15.4)_5$ and $(15.5)_5$ clearly reflect such conclusions.

We should point out finally that table $XIII_5$, summary of this solution, indicates to us that the differences $|T - T_t|$, both in the PKiKP and PKIKP waves are always inferior to 1.4 sec.

16. FINAL OBSERVATIONS

It seems appropriate to conclude this work by noting some observations on the results obtained.

Evidently, all points D"-cusp. detected are not possible, since logically these points must be situated in the interval of values (Δ = 152°.24, Δ = 110°) belonging to solutions 1 and 4. Possibly, there may exist a value that optimizes the results obtained, but we are unable to decide on its existence without additional data observed and registered.

On the other hand, all solutions obtained depend on the value Δ = 121° ± 1° assigned to the point D-cusp. (Choy-Cormier), and any one variation of this value determine modifications in the solution. Depending on the value chosen, the point D-cusp. will appear in real o virtual form according to the solution D'-cusp and D"-cusp. adopted.

On the basis of the condition $k'_2 \neq k_3$, an alternative solution to the problem can be obtained by means of a system of 13 independent equations with 13 unknown quantities, once we establish the value of the minimum $E_0(\Delta, T)$ of the inner core, to obtain at the same time the parameters corresponding to the transition zone and inner core.

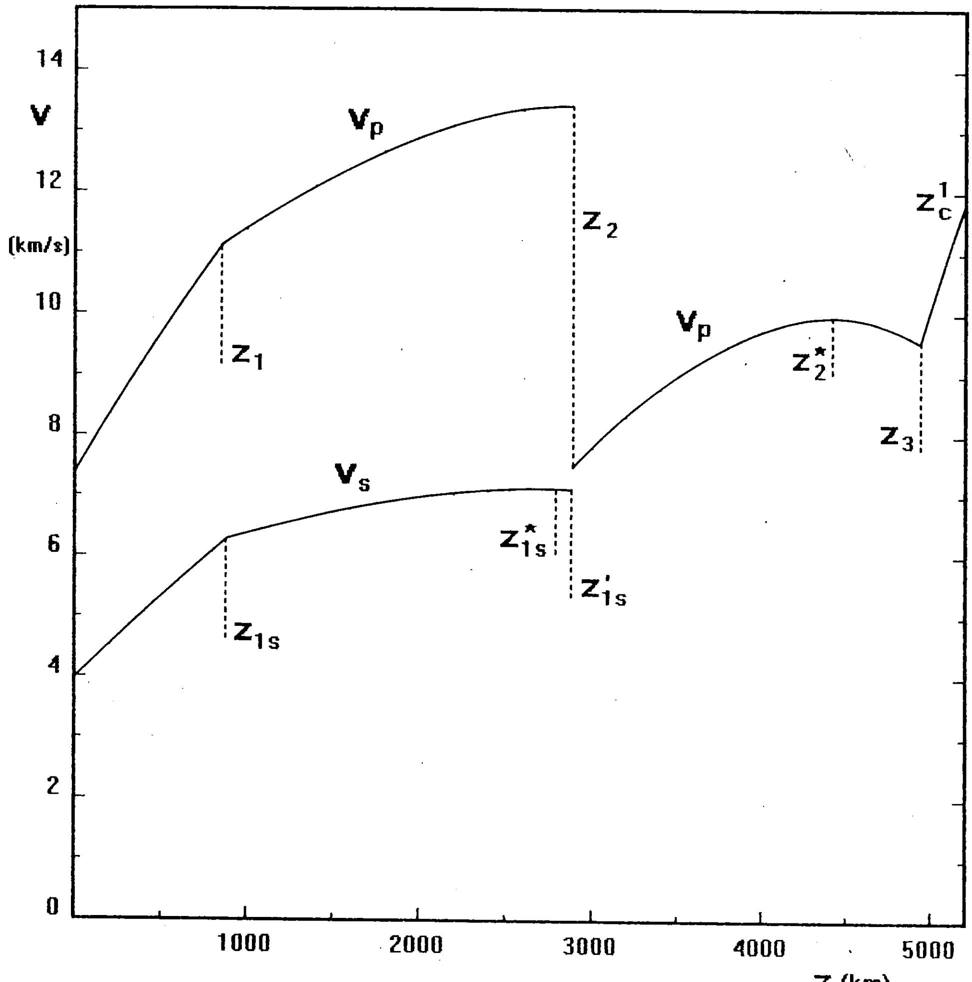
In this manner we should obtain the values k'_2 , of the outer core, the values A_3 , k_3 , k'_3 of the transition zone and the value k_c of the inner core.

With these values, we can calculate the remaining unknown parameters.

Finally, it may be interesting to include the following table:

	r (km)	z (km)	V_p (km/s)	V_s (km/s)
	R_0 6371.0280	0.0000	7.3969	3.9668
U	6271.0280	100.0000	7.8677	4.2478
P	6171.0280	200.0000	8.3291	4.5232
P	6071.0280	300.0000	8.7808	4.7930
E	5971.0280	400.0000	9.2229	5.0571
R	5871.0280	500.0000	9.6550	5.3154
	5771.0280	600.0000	10.0770	5.5678
Mantle	5671.0280	700.0000	10.4888	5.8142
	5571.0280	800.0000	10.8902	6.0546
	R_1 5525.2036	845.8244	11.0705	6.1626
	R_{1s} 5487.7504	883.2776	11.1455	6.2500
	5471.0280	900.0000	11.1787	6.2642
LOWER	5371.0280	1000.0000	11.3727	6.3470
	5271.0280	1100.0000	11.5592	6.4261
Mantle	5171.0280	1200.0000	11.7380	6.5014
	5071.0280	1300.0000	11.9091	6.5729
	4971.0280	1400.0000	12.0723	6.6405

	r (km)	z (km)	v_p (km/s)	v_s (km/s)
	4871.0280	1500.0000	12.2273	6.7040
	4771.0280	1600.0000	12.3742	6.7635
L	4671.0280	1700.0000	12.5125	6.8187
O	4571.0280	1800.0000	12.6423	6.8698
W	4471.0280	1900.0000	12.7633	6.9164
E	4371.0280	2000.0000	12.8753	6.9586
R	4271.0280	2100.0000	12.9781	6.9963
	4171.0280	2200.0000	13.0715	7.0293
	4071.0280	2300.0000	13.1552	7.0575
M	3971.0280	2400.0000	13.2291	7.0808
A	3871.0280	2500.0000	13.2929	7.0991
N	3771.0280	2600.0000	13.3462	7.1123
T	3671.0280	2700.0000	13.3889	7.1201
L	r^*_{1s} 3576.5939	2794.4341	13.2568	7.1226
E	3571.0280	2800.0000	13.4207	7.1226
	R'_{1s} 3486.2571	2884.7709	13.4388	7.1203
	R'_1 3477.7619	2893.2661	13.4401	
	R_2 3477.7619	2893.2661	7.4804	
	3471.0280	2900.0000	7.5003	
O	3371.0280	3000.0000	7.7874	
U	3271.0280	3100.0000	8.0595	
T	3171.0280	3200.0000	8.3159	
E	3071.0280	3300.0000	8.5562	
R	2971.0280	3400.0000	8.7799	
	2871.0280	3500.0000	8.9863	
	2771.0280	3600.0000	9.1750	
C	2671.0280	3700.0000	9.3453	
O	2571.0280	3800.0000	9.4964	
R	2471.0280	3900.0000	9.6277	
E	2371.0280	4000.0000	9.7383	
	2271.0280	4100.0000	9.8273	
	2171.0280	4200.0000	9.8939	



V
(km/s)
0
2
4
6
8
10
12
14
Vp
Vp
Vs
Z1
Z2
Z1s
Z*1s
Z'1s
Z*2
Z3
Z1c
1000
2000
3000
4000
5000
Z (km)

Fig. 16.1

	r (km)	z (km)	v_p (km/s)	v_s (km/s)
	2071.0280	4300.0000	9.9369	
O	1971.0280	4400.0000	9.9552	
U	r_2^* 1949.4940	4421.5340	9.9558	
T	1871.0280	4500.0000	9.9477	
E	1771.0280	4600.0000	9.9128	
R	1671.0280	4700.0000	9.8491	
	1571.0280	4800.0000	9.7547	
CORE	1471.0280	4900.0000	9.6279	
	R_3 1428.1277	4942.9003	9.5630	
	1371.0280	5000.0000	10.1227	
Tran	1271.0280	5100.0000	11.0054	
sition	R_c^4 1266.5286	5104.4994	11.0421	
	R_c^3 1229.6681	5141.3599	11.3323	
ZO	R_c^2 1216.0000	5155.0280	11.4352	
NE	R_c^5 1196.3902	5174.6378	11.5784	
	R_c^1 1174.9196	5196.1084	11.7289	

that include the velocities v_p and v_s in function of the radius r.

The values $v_p(r)$ and $v_s(r)$ have been obtained by means of the formula $v = r\,(B_i - A_i \log r)$, where the parameters A_i , B_i , correspond to the values of each zone.

In this context, we have summarized the velocities $v_p(z)$ and $v_s(z)$ in Fig. 16.1 . where the maxima of the velocities have been denoted by the symbol (*).

17. TABLES

In general the tables included in this work contain the following data:

I_o = initial angle

$\Delta°$ = epicentral distance in sexagesimal degrees

T = travel-time in seconds

T_t = travel-time in seconds of standard tables

$T - T_t$ = difference between both travel-times

In some tables we have included, moreover, the data corresponding to the P_m point (middle point or deepest point of the trajectory), i.e.:

$$r_m = r(P_m)$$

$$z_m = R_0 - r_m$$

$$v_m = v(r_m)$$

$$k_m = k(r_m)$$

Exceptionally, table XII_1 contains the data I_0, I_3', I_c, T_E, k_3', k_c, R_c, v_3', v_c and table XII_2 the data R_c, I_0, $\Delta°$, T, for the points D'-cusp. and D"-cusp., and for an epicentral distance $\Delta° = 146°$.

We now turn our attention to the formulation used in the elaboration of these tables, and any results that have not been sufficiently explained:

a) In table I, for the P waves of the upper mantle we observe that the travel-times obtained present a maximum difference of the order $|T - T_t| = 4.8372$ sec. for $\Delta° = 2°$. This difference is probably due to the irregularities of the continental crust and Mohorovicic discontinuity, since for epicentral distances $\Delta° > 4°$, these differences are appreciatively smaller.

The problem is emphasized for the S waves. In this case the model established probably requires some modification.

b) The formulation employed in the computation of the quantities $k(r_m)$, r_m, z_m, v_m, has been the following:

$$k(r_m) = \frac{\sin I(r_m)}{p_0} = \frac{\sin 90°}{p_0} = \frac{1}{p_0} \qquad (17.1)$$

$$r_m = R_i \exp\left[\frac{k_i - k(r_m)}{A_i}\right] \tag{17.2}$$

$$z_m = R_0 - r_m \tag{17.3}$$

$$v_m' = k(r_m)\, r_m \tag{17.4}$$

In particular, for any point $P(r)$ of the inner core, we have $v(r) = v_c$ and therefore

$$p_0 = \frac{R_c \sin I_c}{v_c} = \frac{r \sin I(r)}{v_c}$$

from where the formula

$$r_m = R_c \sin I_c$$

substitutes the equality (17.2) in the calculation of the radius r_m in this case.

c) In table VII (PcS waves) we have considered a unique radius R_2 (corresponding to P waves) for the computation of the waves reflected.

In this case we have utilized the value

$$k_{1s}' = k_{1s} - A_{1s} \log \frac{R_2}{R_{1s}} = 2.04725450556 \times 10^{-3}$$

deduced from the formula

$$R_2 = R_{1s} \exp\left[\frac{k_{1s} - k_{1s}'}{A_{1s}}\right]$$

d) Finally, in table X ($PKP^{(T)}$ waves) in order to obtain the value I_0 corresponding to the radius R_c = 1216 Km., we have proceeded in the following way:

$$R_c = R_3 \exp\left[\frac{k_3 - k}{A_3}\right]$$

from where

$$k = k_3 - A_3 \log \frac{R_c}{R_3} = 9.40396865058 \times 10^{-3}$$

and therefore

$$I_0 = \arcsin \frac{k_0}{k} = 7^\circ.09190363064$$

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TABLES

TABLE I P WAVES UPPER MANTLE

I_0	$\Delta°$	T (sec.)	T_t (sec.)	$T - T_t$
90.0000	0	0.0000	0.0000	0.0000
87.4541	1	15.0277	18.5323	-3.5046
84.9182	2	30.0258	34.8630	-4.8372
82.4022	3	44.9653	48.5813	-3.6160
79.9152	4	59.8180	62.2906	-2.4726
77.4660	5	74.5575	75.9880	-1.4305
75.0625	6	89.1589	89.6703	-0.5114
72.7116	7	103.5996	103.3346	0.2650
70.4192	8	117.8595	116.9779	0.8816
68.1902	9	131.9208	130.5973	1.3235
66.0284	10	145.7682	144.1896	1.5786
63.9369	11	159.3890	157.7519	1.6371
61.9174	12	172.7731	171.2813	1.4918
59.9711	13	185.9124	184.7746	1.1378
58.0983	14	198.8012	198.1926	0.6086
56.2987	15	211.4356	211.4756	-0.0400
54.5714	16	223.8133	224.5485	-0.7352
52.9152	17	235.9340	237.3414	-1.4074
51.3283	18	247.7982	249.7793	-1.9811
49.8087	19	259.4079	261.7872	-2.3793
48.3543	20	270.7660	273.3185	-2.5525
46.9628	21	281.8761	284.3693	-2.4932
45.6315	22	292.7426	294.9501	-2.2075
44.3582	23	303.3702	305.1134	-1.7432
43.1402	24	313.7643	314.9070	-1.1427
41.9751	25	323.9304	324.3869	-0.4565
40.8603	26	333.8741	333.6295	0.2446
39.7936	27	343.6016	342.7068	0.8948
38.7724	28	353.1186	351.6796	1.4390
37.7947	29	362.4313	360.6048	1.8265
36.8582	30	371.5457	369.5086	2.0371
35.9608	31	380.4676	378.3751	2.0925
35.1005	32	389.2029	387.1923	2.0106
34.2755	33	397.7574	395.9621	1.7953
33.4840	34	406.1367	404.6807	1.4560
32.7241	35	414.3463	413.3435	1.0028
31.9944	36	422.3916	421.9479	0.4437
31.2933	37	430.2778	430.4894	-0.2116
30.6193	38	438.0098	438.9626	-0.9528
29.9711	39	445.5926	447.3662	-1.7736
29.3473	40	453.0308	455.7020	-2.6712

TABLE II S WAVES UPPER MANTLE

I_0	$\Delta°$	T (sec.)	T_t (sec.)	$T - T_t$
90.0000	0	0.0000	0.00	0.0000
87.2243	1	28.0203	32.71	-4.6897
84.4616	2	55.9750	62.32	-6.3450
81.7245	3	83.8000	88.11	-4.3100
79.0248	4	111.4337	113.46	-2.0263
76.3736	5	138.8185	138.82	-0.0015
73.7805	6	165.9014	164.17	1.7314
71.2539	7	192.6347	189.52	3.1147
68.8006	8	218.9769	214.88	4.0969
66.4262	9	244.8925	240.23	4.6625
64.1347	10	270.3519	265.58	4.7719
61.9286	11	295.3318	290.91	4.4218
59.8095	12	319.8142	316.09	3.7242
57.7778	13	343.7864	341.28	2.5064
55.8330	14	367.2404	366.36	0.8804
53.9738	15	390.1721	391.16	-0.9879
52.1984	16	412.5812	415.67	-3.0888
50.5045	17	434.4705	439.57	-5.0995
48.8892	18	455.8453	462.53	-6.6847
47.3496	19	476.7130	484.01	-7.2970
45.8827	20	497.0826	503.94	-6.8573
44.4851	21	516.9648	522.94	-5.9752
43.1537	22	536.3709	541.33	-4.9591
41.8851	23	555.3133	559.11	-3.7967
40.6762	24	573.8047	575.91	-2.1053
39.5239	25	591.8583	592.37	-0.5117
38.4251	26	609.4873	608.55	0.9373
37.3770	27	626.7051	624.55	2.1551
36.3768	28	643.5248	640.44	3.0848
35.4219	29	659.9597	656.20	3.7597
34.5097	30	676.0225	671.87	4.1525
33.6379	31	691.7258	687.42	4.3058
32.8042	32	707.0819	702.90	4.1819
32.0065	33	722.1027	718.29	3.8127
31.2429	34	736.7997	733.60	3.1997
30.5114	35	751.1841	748.85	2.3341
29.8102	36	765.2666	764.05	1.2166
29.1378	37	779.0577	779.21	-0.1523
28.4926	38	792.5674	794.34	-1.7726
27.8730	39	805.8052	809.39	-3.5848
27.2778	40	818.7803	824.37	-5.5897

TABLE III P WAVES

I_0	$\Delta°$	T (sec.)	T_t (sec.)	$T - T_t$
90.0000	0.0000	0.0000	0.0000	0.0000
87.4541	1.0000	15.0277	18.5323	-3.5046
84.9182	2.0000	30.0258	34.8630	-4.8372
82.4022	3.0000	44.9653	48.5813	-3.6160
79.9152	4.0000	59.8180	62.2906	-2.4726
77.4660	5.0000	74.5575	75.9880	-1.4305
75.0625	6.0000	89.1589	89.6703	-0.5114
72.7116	7.0000	103.5996	103.3346	0.2650
70.4192	8.0000	117.8595	116.9779	0.8816
68.1902	9.0000	131.9208	130.5973	1.3235
66.0284	10.0000	145.7682	144.1896	1.5786
63.9369	11.0000	159.3890	157.7519	1.6371
61.9174	12.0000	172.7731	171.2813	1.4918
59.9711	13.0000	185.9124	184.7746	1.1378
58.0983	14.0000	198.8012	198.1926	0.6086
56.2987	15.0000	211.4356	211.4756	-0.0400
54.5714	16.0000	223.8133	224.5485	-0.7352
52.9152	17.0000	235.9340	237.3414	-1.4074
51.3283	18.0000	247.7982	249.7793	-1.9811
49.8087	19.0000	259.4079	261.7872	-2.3793
48.3543	20.0000	270.7660	273.3185	-2.5525
46.9628	21.0000	281.8761	284.3693	-2.4932
45.6315	22.0000	292.7426	294.9501	-2.2075
44.3582	23.0000	303.3702	305.1134	-1.7432
43.1402	24.0000	313.7643	314.9070	-1.1427
41.9751	25.0000	323.9304	324.3869	-0.4565
40.8603	26.0000	333.8741	333.6295	0.2446
39.7936	27.0000	343.6016	342.7068	0.8948
38.7724	28.0000	353.1186	351.6796	1.4390
37.7947	29.0000	362.4313	360.6048	1.8265
36.8582	30.0000	371.5457	369.5086	2.0371
35.9608	31.0000	380.4676	378.3751	2.0925
35.4124	31.6326	386.0153	383.9579	2.0574
35.4046	32.0000	389.2150	387.1923	2.0227
35.3169	33.0000	397.9163	395.9621	1.9542

TABLE III P WAVES

$\Delta°$	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
0.0000	1.1610	6371.0	0.0	7.3969
1.0000	1.1622	6369.8	1.2	7.4028
2.0000	1.1656	6366.1	4.9	7.4204
3.0000	1.1713	6360.0	11.0	7.4495
4.0000	1.1792	6351.4	19.6	7.4899
5.0000	1.1894	6340.6	30.4	7.5413
6.0000	1.2016	6327.4	43.6	7.6033
7.0000	1.2160	6312.1	58.9	7.6753
8.0000	1.2323	6294.7	76.3	7.7569
9.0000	1.2505	6275.3	95.7	7.8476
10.0000	1.2706	6254.1	116.9	7.9466
11.0000	1.2925	6231.0	140.0	8.0533
12.0000	1.3160	6206.3	164.7	8.1672
13.0000	1.3410	6180.1	190.9	8.2877
14.0000	1.3676	6152.4	218.6	8.4140
15.0000	1.3956	6123.4	247.6	8.5455
16.0000	1.4248	6093.1	277.9	8.6818
17.0000	1.4554	6061.7	309.3	8.8222
18.0000	1.4871	6029.4	341.6	8.9662
19.0000	1.5199	5996.0	375.0	9.1132
20.0000	1.5537	5961.9	409.1	9.2629
21.0000	1.5885	5926.9	444.1	9.4147
22.0000	1.6241	5891.3	479.7	9.5682
23.0000	1.6606	5855.0	516.0	9.7231
24.0000	1.6979	5818.2	552.8	9.8790
25.0000	1.7360	5781.0	590.0	10.0355
26.0000	1.7747	5743.2	627.8	10.1924
27.0000	1.8140	5705.2	665.8	10.3494
28.0000	1.8540	5666.7	704.3	10.5061
29.0000	1.8945	5628.1	742.9	10.6625
30.0000	1.9356	5589.1	781.9	10.8182
31.0000	1.9771	5550.0	821.0	10.9731
31.6326	2.0036	5525.2	845.8	11.0705
32.0000	2.0040	5524.7	846.3	11.0716
33.0000	2.0083	5518.7	852.3	11.0836

TABLE III P WAVES

I_0	$\Delta°$	T (sec.)	T_t (sec.)	$T - T_t$
35.1558	34.0000	406.5905	404.6807	1.9098
34.9417	35.0000	415.2241	413.3435	1.8806
34.6883	36.0000	423.8073	421.9479	1.8594
34.4054	37.0000	432.3325	430.4894	1.8431
34.1000	38.0000	440.7938	438.9626	1.8312
33.7776	39.0000	449.1869	447.3662	1.8207
33.4425	40.0000	457.5083	455.7020	1.8062
33.0979	41.0000	465.7551	463.9710	1.7841
32.7466	42.0000	473.9255	472.1723	1.7532
32.3907	43.0000	482.0178	480.3051	1.7127
32.0320	44.0000	490.0308	488.3680	1.6628
31.6719	45.0000	497.9640	496.3596	1.6044
31.3115	46.0000	505.8166	504.2791	1.5375
30.9520	47.0000	513.5886	512.1242	1.4644
30.5941	48.0000	521.2798	519.8920	1.3878
30.2385	49.0000	528.8905	527.5828	1.3077
29.8857	50.0000	536.4208	535.1992	1.2216
29.5362	51.0000	543.8713	542.7433	1.1280
29.1904	52.0000	551.2424	550.2164	1.0260
28.8486	53.0000	558.5348	557.6192	0.9156
28.5111	54.0000	565.7491	564.9510	0.7981
28.1780	55.0000	572.8861	572.2107	0.6754
27.8495	56.0000	579.9466	579.3986	0.5480
27.5257	57.0000	586.9314	586.5135	0.4179
27.2068	58.0000	593.8415	593.5538	0.2877
26.8928	59.0000	600.6777	600.5205	0.1572
26.5837	60.0000	607.4410	607.4162	0.0248
26.2795	61.0000	614.1324	614.2444	-0.1120
25.9803	62.0000	620.7528	621.0084	-0.2556
25.6861	63.0000	627.3032	627.7094	-0.4062
25.3968	64.0000	633.7846	634.3452	-0.5606
25.1123	65.0000	640.1980	640.9137	-0.7157
24.8328	66.0000	646.5445	647.4142	-0.8697
24.5580	67.0000	652.8249	653.8477	-1.0228
24.2880	68.0000	659.0404	660.2151	-1.1747

TABLE III P WAVES

Δ°	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
34.0000	2.0164	5507.7	863.3	11.1056
35.0000	2.0271	5493.0	878.0	11.1351
36.0000	2.0401	5475.4	895.6	11.1701
37.0000	2.0547	5455.4	915.6	11.2095
38.0000	2.0709	5433.5	937.5	11.2523
39.0000	2.0883	5410.1	960.9	11.2978
40.0000	2.1067	5385.3	985.7	11.3454
41.0000	2.1261	5359.4	1011.6	11.3948
42.0000	2.1464	5332.5	1038.5	11.4455
43.0000	2.1673	5304.7	1066.3	11.4972
44.0000	2.1890	5276.2	1094.8	11.5496
45.0000	2.2112	5247.1	1123.9	11.6027
46.0000	2.2341	5217.4	1153.6	11.6560
47.0000	2.2574	5187.2	1183.8	11.7096
48.0000	2.2812	5156.6	1214.4	11.7632
49.0000	2.3054	5125.6	1245.4	11.8168
50.0000	2.3301	5094.2	1276.8	11.8701
51.0000	2.3551	5062.6	1308.4	11.9232
52.0000	2.3805	5030.7	1340.3	11.9758
53.0000	2.4063	4998.6	1372.4	12.0281
54.0000	2.4323	4966.3	1404.7	12.0798
55.0000	2.4587	4933.8	1437.2	12.1309
56.0000	2.4853	4901.3	1469.7	12.1813
57.0000	2.5122	4868.6	1502.4	12.2310
58.0000	2.5394	4835.8	1535.2	12.2800
59.0000	2.5668	4802.9	1568.1	12.3282
60.0000	2.5944	4770.0	1601.0	12.3756
61.0000	2.6223	4737.1	1633.9	12.4221
62.0000	2.6504	4704.1	1666.9	12.4677
63.0000	2.6786	4671.2	1699.8	12.5123
64.0000	2.7071	4638.2	1732.8	12.5561
65.0000	2.7357	4605.3	1765.7	12.5988
66.0000	2.7645	4572.4	1798.6	12.6406
67.0000	2.7935	4539.5	1831.5	12.6813
68.0000	2.8227	4506.8	1864.2	12.7211

TABLE III P WAVES

I_0	$\Delta°$	T (sec.)	T_t (sec.)	$T - T_t$
24.0226	69.0000	665.1918	666.5134	-1.3216
23.7619	70.0000	671.2802	672.7383	-1.4581
23.5057	71.0000	677.3065	678.8805	-1.5740
23.2541	72.0000	683.2718	684.9366	-1.6648
23.0068	73.0000	689.1769	690.9092	-1.7323
22.7639	74.0000	695.0229	696.8054	-1.7825
22.5253	75.0000	700.8105	702.6299	-1.8194
22.2908	76.0000	706.5409	708.3843	-1.8434
22.0604	77.0000	712.2148	714.0661	-1.8513
21.8341	78.0000	717.8333	719.6690	-1.8357
21.6117	79.0000	723.3970	725.1920	-1.7950
21.3932	80.0000	728.9070	730.6349	-1.7279
21.1786	81.0000	734.3641	735.9998	-1.6356
20.9676	82.0000	739.7692	741.2871	-1.5179
20.7603	83.0000	745.1230	746.4926	-1.3696
20.5565	84.0000	750.4264	751.6058	-1.1794
20.3563	85.0000	755.6801	756.6260	-0.9459
20.1594	86.0000	760.8850	761.5636	-0.6786
19.9660	87.0000	766.0419	766.4338	-0.3919
19.7758	88.0000	771.1514	771.2455	-0.0941
19.5888	89.0000	776.2144	776.0056	0.2088
19.4050	90.0000	781.2316	780.7222	0.5094
19.2243	91.0000	786.2037	785.4049	0.7988
19.0466	92.0000	791.1314	790.0597	1.0717
18.8718	93.0000	796.0153	794.6891	1.3262
18.6999	94.0000	800.8562	799.2966	1.5596
18.5308	95.0000	805.6548	803.8872	1.7676
18.3645	96.0000	810.4117	808.4658	1.9459
18.2010	97.0000	815.1274	813.0361	2.0913
18.0400	98.0000	819.8028	817.6016	2.2012
17.8817	99.0000	824.4383	822.1660	2.2723
17.7259	100.0000	829.0346	826.7303	2.3043
17.5725	101.0000	833.5923	831.2946	2.2977
17.4832	101.5901	836.2640	833.9881	2.2759

TABLE III P WAVES

$\Delta°$	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
69.0000	2.8520	4474.0	1897.0	12.7598
70.0000	2.8814	4441.4	1929.6	12.7975
71.0000	2.9110	4408.8	1962.2	12.8341
72.0000	2.9407	4376.3	1994.7	12.8696
73.0000	2.9706	4343.9	2027.1	12.9041
74.0000	3.0006	4311.7	2059.3	12.9375
75.0000	3.0307	4279.5	2091.5	12.9698
76.0000	3.0609	4247.4	2123.6	13.0010
77.0000	3.0912	4215.5	2155.5	13.0311
78.0000	3.1217	4183.7	2187.3	13.0602
79.0000	3.1523	4152.0	2219.0	13.0882
80.0000	3.1829	4120.4	2250.6	13.1151
81.0000	3.2137	4089.0	2282.0	13.1409
82.0000	3.2445	4057.8	2313.2	13.1656
83.0000	3.2755	4026.6	2344.4	13.1893
84.0000	3.3065	3995.7	2375.3	13.2118
85.0000	3.3377	3964.8	2406.2	13.2333
86.0000	3.3689	3934.2	2436.8	13.2538
87.0000	3.4002	3903.7	2467.3	13.2732
88.0000	3.4315	3873.3	2497.7	13.2915
89.0000	3.4630	3843.2	2527.8	13.3088
90.0000	3.4945	3813.1	2557.9	13.3250
91.0000	3.5261	3783.3	2587.7	13.3402
92.0000	3.5578	3753.6	2617.4	13.3544
93.0000	3.5895	3724.1	2646.9	13.3676
94.0000	3.6213	3694.7	2676.3	13.3798
95.0000	3.6531	3665.6	2705.4	13.3909
96.0000	3.6851	3636.6	2734.4	13.4011
97.0000	3.7171	3607.8	2763.2	13.4103
98.0000	3.7491	3579.1	2791.9	13.4185
99.0000	3.7812	3550.6	2820.4	13.4258
100.0000	3.8134	3522.4	2848.6	13.4321
101.0000	3.8456	3494.3	2876.7	13.4374
101.5901	3.8646	3477.7	2893.3	13.4401

TABLE IV S WAVES

I_0	Δ°	T (sec.)	T_t (sec.)	$T - T_t$
90.0000	0.0000	0.0000	0.00	0.0000
87.2243	1.0000	28.0203	32.71	-4.6897
84.4616	2.0000	55.9750	62.32	-6.3450
81.7245	3.0000	83.8000	88.11	-4.3100
79.0248	4.0000	111.4337	113.46	-2.0263
76.3736	5.0000	138.8185	138.82	-0.0015
73.7805	6.0000	165.9014	164.17	1.7314
71.2539	7.0000	192.6347	189.52	3.1147
68.8006	8.0000	218.9769	214.88	4.0969
66.4262	9.0000	244.8925	240.23	4.6625
64.1347	10.0000	270.3519	265.58	4.7719
61.9286	11.0000	295.3318	290.91	4.4218
59.8095	12.0000	319.8142	316.09	3.7242
57.7778	13.0000	343.7864	341.28	2.5064
55.8330	14.0000	367.2404	366.36	0.8804
53.9738	15.0000	390.1721	391.16	-0.9879
52.1984	16.0000	412.5812	415.67	-3.0888
50.5045	17.0000	434.4705	439.57	-5.0995
48.8892	18.0000	455.8453	462.53	-6.6847
47.3496	19.0000	476.7130	484.01	-7.2970
45.8827	20.0000	497.0826	503.94	-6.8573
44.4851	21.0000	516.9648	522.94	-5.9752
43.1537	22.0000	536.3709	541.33	-4.9591
41.8851	23.0000	555.3133	559.11	-3.7967
40.6762	24.0000	573.8047	575.91	-2.1053
39.5239	25.0000	591.8583	592.37	-0.5117
38.4251	26.0000	609.4873	608.55	0.9373
37.3770	27.0000	626.7051	624.55	2.1551
36.3768	28.0000	643.5248	640.44	3.0848
35.4219	29.0000	659.9597	656.20	3.7597
34.5097	30.0000	676.0225	671.87	4.1525
33.6379	31.0000	691.7258	687.42	4.3058
33.1411	31.5905	700.8353	696.57	4.2657
33.1371	32.0000	707.1100	702.90	4.2100
33.0967	33.0000	722.4263	718.29	4.1363
33.0181	34.0000	737.7180	733.60	4.1180

TABLE IV S WAVES

Δ°	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
0.0000	0.6226	6371.0	0.0	3.9668
1.0000	0.6234	6369.7	1.3	3.9707
2.0000	0.6256	6365.6	5.4	3.9821
3.0000	0.6292	6359.0	12.0	4.0010
4.0000	0.6342	6349.7	21.3	4.0272
5.0000	0.6407	6337.9	33.1	4.0605
6.0000	0.6484	6323.7	47.3	4.1006
7.0000	0.6575	6307.1	63.9	4.1470
8.0000	0.6678	6288.3	82.7	4.1995
9.0000	0.6793	6267.4	103.6	4.2577
10.0000	0.6920	6244.6	126.4	4.3210
11.0000	0.7056	6220.0	151.0	4.3891
12.0000	0.7203	6193.6	177.4	4.4615
13.0000	0.7360	6165.6	205.4	4.5378
14.0000	0.7525	6136.2	234.8	4.6176
15.0000	0.7699	6105.5	265.5	4.7005
16.0000	0.7880	6073.6	297.4	4.7861
17.0000	0.8069	6040.6	330.4	4.8739
18.0000	0.8264	6006.6	364.4	4.9638
19.0000	0.8465	5971.7	399.3	5.0553
20.0000	0.8673	5936.0	435.0	5.1482
21.0000	0.8886	5899.6	471.4	5.2422
22.0000	0.9103	5862.6	508.4	5.3370
23.0000	0.9326	5825.0	546.0	5.4324
24.0000	0.9553	5786.9	584.1	5.5281
25.0000	0.9784	5748.4	622.6	5.6241
26.0000	1.0018	5709.5	661.5	5.7201
27.0000	1.0257	5670.3	700.7	5.8159
28.0000	1.0498	5630.9	740.1	5.9114
29.0000	1.0743	5591.2	779.8	6.0065
30.0000	1.0990	5551.4	819.6	6.1010
31.0000	1.1240	5511.4	859.6	6.1949
31.5905	1.1389	5487.7	883.3	6.2500
32.0000	1.1390	5487.4	883.6	6.2503
33.0000	1.1402	5484.0	887.0	6.2531
34.0000	1.1426	5477.4	893.6	6.2588

TABLE IV S WAVES

I_0	$\Delta°$	T (sec.)	T_t (sec.)	$T - T_t$
32.9072	35.0000	752.9706	748.85	4.1206
32.7688	36.0000	768.1718	764.05	4.1218
32.6070	37.0000	783.3112	779.21	4.1012
32.4253	38.0000	798.3797	794.34	4.0397
32.2267	39.0000	813.3695	809.39	3.9795
32.0136	40.0000	828.2741	824.37	3.9041
31.7884	41.0000	843.0876	839.28	3.8076
31.5529	42.0000	857.8053	854.13	3.6753
31.3089	43.0000	872.4230	868.91	3.5130
31.0577	44.0000	886.9372	883.57	3.3672
30.8008	45.0000	901.3448	898.11	3.2348
30.5391	46.0000	915.6435	912.53	3.1135
30.2738	47.0000	929.8311	926.79	3.0411
30.0056	48.0000	943.9059	941.00	2.9059
29.7354	49.0000	957.8667	955.15	2.7167
29.4638	50.0000	971.7123	969.31	2.4023
29.1914	51.0000	985.4421	983.37	2.0721
28.9188	52.0000	999.0555	997.32	1.7355
28.6464	53.0000	1012.5521	1011.18	1.3721
28.3746	54.0000	1025.9319	1024.98	0.9519
28.1037	55.0000	1039.1950	1038.56	0.6350
27.8342	56.0000	1052.3414	1051.80	0.5414
27.5662	57.0000	1065.3715	1064.82	0.5515
27.3000	58.0000	1078.2857	1077.74	0.5457
27.0359	59.0000	1091.0847	1090.61	0.4747
26.7739	60.0000	1103.7691	1103.42	0.3491
26.5143	61.0000	1116.3395	1116.20	0.1395
26.2572	62.0000	1128.7968	1128.92	-0.1232
26.0026	63.0000	1141.1419	1141.55	-0.4081
25.7508	64.0000	1153.3756	1154.10	-0.7244
25.5018	65.0000	1165.4990	1166.58	-1.0810
25.2556	66.0000	1177.5131	1178.95	-1.4369
25.0123	67.0000	1189.4188	1191.20	-1.7812
24.7720	68.0000	1201.2174	1203.33	-2.1126
24.5346	69.0000	1212.9098	1215.32	-2.4102
24.3002	70.0000	1224.4973	1227.13	-2.6327

TABLE IV S WAVES

$\Delta°$	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
35.0000	1.1461	5468.0	903.0	6.2667
36.0000	1.1504	5456.2	914.8	6.2767
37.0000	1.1554	5442.3	928.7	6.2883
38.0000	1.1612	5426.6	944.4	6.3014
39.0000	1.1676	5409.2	961.8	6.3158
40.0000	1.1745	5390.4	980.6	6.3312
41.0000	1.1820	5370.3	1000.7	6.3475
42.0000	1.1899	5349.1	1021.9	6.3647
43.0000	1.1982	5326.8	1044.2	6.3825
44.0000	1.2069	5303.5	1067.5	6.4008
45.0000	1.2160	5279.4	1091.6	6.4196
46.0000	1.2254	5254.6	1116.4	6.4387
47.0000	1.2351	5229.0	1142.0	6.4582
48.0000	1.2451	5202.8	1168.2	6.4779
49.0000	1.2553	5176.1	1194.9	6.4977
50.0000	1.2658	5148.8	1222.2	6.5176
51.0000	1.2766	5121.1	1249.9	6.5376
52.0000	1.2876	5092.9	1278.1	6.5576
53.0000	1.2988	5064.4	1306.6	6.5775
54.0000	1.3102	5035.5	1335.5	6.5974
55.0000	1.3217	5006.3	1364.7	6.6171
56.0000	1.3335	4976.8	1394.2	6.6367
57.0000	1.3454	4947.1	1423.9	6.6560
58.0000	1.3575	4917.1	1453.9	6.6752
59.0000	1.3698	4887.0	1484.0	6.6941
60.0000	1.3822	4856.6	1514.4	6.7128
61.0000	1.3947	4826.1	1544.9	6.7312
62.0000	1.4074	4795.5	1575.5	6.7493
63.0000	1.4202	4764.8	1606.2	6.7670
64.0000	1.4331	4734.0	1637.0	6.7844
65.0000	1.4462	4703.1	1667.9	6.8015
66.0000	1.4593	4672.1	1698.9	6.8182
67.0000	1.4726	4641.1	1729.9	6.8345
68.0000	1.4860	4610.0	1761.0	6.8504
69.0000	1.4994	4578.9	1792.1	6.8659
70.0000	1.5130	4547.8	1823.2	6.8810

TABLE IV S WAVES

I_0	Δ°	T (sec.)	T_t (sec.)	$T - T_t$
24.0689	71.0000	1235.9809	1238.76	-2.7791
23.8406	72.0000	1247.3619	1250.18	-2.8181
23.6154	73.0000	1258.6414	1261.49	-2.8486
23.3931	74.0000	1269.8206	1272.70	-2.8794
23.1739	75.0000	1280.9007	1283.84	-2.9393
22.9577	76.0000	1291.8829	1294.85	-2.9671
22.7445	77.0000	1302.7684	1305.76	-2.9916
22.5343	78.0000	1313.5584	1316.53	-2.9716
22.3271	79.0000	1324.2540	1327.19	-2.9360
22.1227	80.0000	1334.8565	1337.71	-2.8535
21.9213	81.0000	1345.3671	1348.09	-2.7279
21.7227	82.0000	1355.7869	1358.34	-2.5531
21.5270	83.0000	1366.1170	1368.45	-2.3330
21.3341	84.0000	1376.3588	1378.42	-2.0612
21.1440	85.0000	1386.5133	1388.25	-1.7367
20.9566	86.0000	1396.5816	1397.96	-1.3784
20.7719	87.0000	1406.5650	1407.56	-0.9950
20.5898	88.0000	1416.4644	1417.04	-0.5756
20.4104	89.0000	1426.2811	1426.39	-0.1088
20.2336	90.0000	1436.0162	1435.61	0.4062
20.0594	91.0000	1445.6706	1444.70	0.9706
19.8876	92.0000	1455.2456	1453.66	1.5856
19.7184	93.0000	1464.7421	1462.60	2.1421
19.5515	94.0000	1474.1613	1471.54	2.6213
19.3871	95.0000	1483.5040	1480.48	3.0240
19.2250	96.0000	1492.7714	1489.42	3.3514
19.0652	97.0000	1501.9645	1498.36	3.6045
18.9078	98.0000	1511.0843	1507.30	3.7843
18.7526	99.0000	1520.1317	1516.24	3.8917
18.5995	100.0000	1529.1076	1525.18	3.9276
18.4487	101.0000	1538.0132	(1534.12)	(3.8932)
18.3000	102.0000	1546.8492	(1543.06)	(3.7892)
18.2193	*102.5484*	*1551.6655*	(1547.96)	(3.7055)
18.1534	103.0000	1555.6167	(1552.00)	(3.6167)
18.0088	104.0000	1564.3164	(1560.94)	(3.3764)
17.8663	105.0000	1572.9494	(1569.88)	(3.0694)
17.7495	105.8299	1580.0640	(1577.30)	(2.7646)

TABLE IV S WAVES

$\Delta°$	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
71.0000	1.5267	4516.7	1854.3	6.8956
72.0000	1.5404	4485.6	1885.4	6.9099
73.0000	1.5543	4454.6	1916.4	6.9237
74.0000	1.5682	4423.5	1947.5	6.9370
75.0000	1.5822	4392.5	1978.5	6.9499
76.0000	1.5963	4361.6	2009.4	6.9624
77.0000	1.6104	4330.7	2040.3	6.9744
78.0000	1.6247	4299.8	2071.2	6.9859
79.0000	1.6390	4269.1	2101.9	6.9970
80.0000	1.6533	4238.4	2132.6	7.0076
81.0000	1.6678	4207.8	2163.2	7.0177
82.0000	1.6823	4177.3	2193.7	7.0273
83.0000	1.6968	4146.8	2224.2	7.0365
84.0000	1.7114	4116.5	2254.5	7.0452
85.0000	1.7261	4086.3	2284.7	7.0535
86.0000	1.7409	4056.2	2314.8	7.0613
87.0000	1.7556	4026.2	2344.8	7.0686
88.0000	1.7705	3996.3	2374.7	7.0754
89.0000	1.7854	3966.5	2404.5	7.0817
90.0000	1.8003	3936.9	2434.1	7.0876
91.0000	1.8153	3907.4	2463.7	7.0930
92.0000	1.8303	3878.0	2493.0	7.0980
93.0000	1.8454	3848.7	2522.3	7.1025
94.0000	1.8605	3819.6	2551.4	7.1065
95.0000	1.8757	3790.6	2580.4	7.1101
96.0000	1.8909	3761.8	2609.2	7.1132
97.0000	1.9062	3733.1	2637.9	7.1159
98.0000	1.9214	3704.5	2666.5	7.1181
99.0000	1.9368	3676.1	2694.9	7.1199
100.0000	1.9521	3647.9	2723.1	7.1212
101.0000	1.9675	3619.8	2751.2	7.1221
102.0000	1.9830	3591.8	2779.2	7.1225
102.5484	*1.9914*	*3576.6*	*2794.4*	*7.1226*
103.0000	1.9984	3564.0	2807.0	7.1225
104.0000	2.0139	3536.4	2834.6	7.1222
105.0000	2.0295	3508.9	2862.1	7.1213
105.8299	2.0424	3486.3	2884.8	7.1203

TABLE V PcP WAVES

I_0	$\Delta°$	T (sec.)	T_t (sec.)	$T - T_t$
0.0000	0.0000	511.3000	511.3	0.0000
0.3665	1.0000	511.3481	511.4	-0.0519
0.7328	2.0000	511.4923	511.5	-0.0077
1.0985	3.0000	511.7325	511.7	-0.0325
1.4634	4.0000	512.0686	512.1	-0.0314
1.8272	5.0000	512.5002	512.5	0.0002
2.1897	6.0000	513.0271	513.0	0.0271
2.5506	7.0000	513.6488	513.6	0.0488
2.9097	8.0000	514.3649	514.4	-0.0351
3.2667	9.0000	515.1748	515.2	-0.0252
3.6214	10.0000	516.0779	516.1	-0.0221
3.9735	11.0000	517.0736	517.1	-0.0264
4.3228	12.0000	518.1610	518.1	0.0610
4.6691	13.0000	519.3395	519.3	0.0395
5.0121	14.0000	520.6081	520.6	0.0081
5.3517	15.0000	521.9659	521.9	0.0659
5.6877	16.0000	523.4119	523.4	0.0119
6.0199	17.0000	524.9452	524.9	0.0452
6.3481	18.0000	526.5646	526.5	0.0646
6.6721	19.0000	528.2691	528.2	0.0691
6.9918	20.0000	530.0574	530.0	0.0574
7.3070	21.0000	531.9285	531.9	0.0285
7.6176	22.0000	533.8809	533.8	0.0809
7.9235	23.0000	535.9135	535.8	0.1135
8.2245	24.0000	538.0250	537.9	0.1250
8.5206	25.0000	540.2140	540.1	0.1140
8.8115	26.0000	542.4791	542.4	0.0791
9.0974	27.0000	544.8191	544.7	0.1191
9.3780	28.0000	547.2324	547.1	0.1324
9.6533	29.0000	549.7176	549.6	0.1176
9.9232	30.0000	552.2734	552.1	0.1734
10.1877	31.0000	554.8982	554.7	0.1982
10.4468	32.0000	557.5907	557.4	0.1907
10.7004	33.0000	560.3492	560.2	0.1492

TABLE V PcP WAVES

I_0	$\Delta°$	T (sec.)	T_t (sec.)	$T - T_t$
10.9484	34.0000	563.1725	563.0	0.1725
11.1910	35.0000	566.0589	565.9	0.1589
11.4279	36.0000	569.0071	568.8	0.2071
11.6594	37.0000	572.0154	571.8	0.2154
11.8853	38.0000	575.0826	574.8	0.2826
12.1057	39.0000	578.2070	578.0	0.2070
12.3206	40.0000	581.3872	581.1	0.2872
12.5300	41.0000	584.6219	584.3	0.3219
12.7339	42.0000	587.9094	587.6	0.3094
12.9324	43.0000	591.2485	590.9	0.3485
13.1256	44.0000	594.6376	594.3	0.3376
13.3134	45.0000	598.0754	597.7	0.3754
13.4959	46.0000	601.5605	601.2	0.3605
13.6732	47.0000	605.0915	604.7	0.3915
13.8453	48.0000	608.6670	608.3	0.3670
14.0122	49.0000	612.2857	611.9	0.3857
14.1742	50.0000	615.9462	615.5	0.4462
14.3311	51.0000	619.6473	619.2	0.4473
14.4831	52.0000	623.3877	622.9	0.4877
14.6303	53.0000	627.1661	626.7	0.4661
14.7726	54.0000	630.9812	630.4	0.5812
14.9103	55.0000	634.8318	634.3	0.5318
15.0433	56.0000	638.7168	638.1	0.6168
15.1718	57.0000	642.6349	642.0	0.6349
15.2958	58.0000	646.5849	646.0	0.5849
15.4154	59.0000	650.5658	649.9	0.6658
15.5307	60.0000	654.5764	653.9	0.6764
15.6418	61.0000	658.6156	657.9	0.7156
15.7487	62.0000	662.6823	661.9	0.7823
15.5815	63.0000	666.7755	666.0	0.7755
15.9503	64.0000	670.8942	670.1	0.7942
15.0452	65.0000	675.0372	674.2	0.8372
16.1363	66.0000	679.2038	678.3	0.9038
16.2236	67.0000	683.3928	682.5	0.8928
16.3073	68.0000	687.6033	686.6	1.0033

TABLE V PcP WAVES

I_0	$\Delta°$	T (sec.)	T_t (sec.)	$T - T_t$
16.3873	69.0000	691.8345	690.8	1.0345
16.4639	70.0000	696.0853	695.1	0.9853
16.5370	71.0000	700.3550	699.3	1.0550
16.6067	72.0000	704.6426	703.5	1.1426
16.6732	73.0000	708.9474	707.9	1.0474
16.7365	74.0000	713.2685	712.1	1.1685
16.7966	75.0000	717.6050	716.5	1.1050
16.8537	76.0000	721.9563	720.7	1.2563
16.9078	77.0000	726.3216	725.0	1.3216
16.9589	78.0000	730.7000	729.3	1.4000
17.0073	79.0000	735.0910	733.6	1.4910
17.0529	80.0000	739.4937	738.0	1.4937
17.0958	81.0000	743.9075	742.4	1.5075
17.1360	82.0000	748.3318	746.7	1.6318
17.1737	83.0000	752.7658	751.1	1.6658
17.2089	84.0000	757.2089	755.5	1.7089
17.2416	85.0000	761.6606	759.9	1.7606
17.2720	86.0000	766.1201	764.3	1.8201
17.3001	87.0000	770.5870	768.7	1.8870
17.3259	88.0000	775.0607	773.1	1.9607
17.3496	89.0000	779.5405	777.5	2.0405
17.3711	90.0000	784.0260	781.9	2.1260
17.3905	91.0000	788.5166	786.3	2.2166
17.4080	92.0000	793.0118	790.8	2.2118
17.4234	93.0000	797.5112	795.2	2.3112
17.4370	94.0000	802.0142	799.6	2.4142
17.4487	95.0000	806.5203	804.0	2.5203
17.4586	96.0000	811.0292	808.5	2.5292
17.4668	97.0000	815.5403		
17.4733	98.0000	820.0532		
17.4781	99.0000	824.5676		
17.4813	100.0000	829.0829		
17.4829	101.0000	833.5989		
17.4832	101.5901	836.2640		

TABLE VI ScS WAVES

I_0	$\Delta°$	T (sec.)	T_t (sec.)	$T - T_t$
0.0000	0.0000	935.7000	935.7	0.0000
0.3649	1.0000	935.7893	935.8	-0.0107
0.7295	2.0000	936.0570	936.1	-0.0430
1.0936	3.0000	936.5029	936.5	0.0029
1.4569	4.0000	937.1268	937.1	0.0268
1.8191	5.0000	937.9281	937.9	0.0281
2.1800	6.0000	938.9062	938.9	0.0062
2.5394	7.0000	940.0605	940.1	-0.0395
2.8970	8.0000	941.3899	941.4	-0.0101
3.2525	9.0000	942.8935	942.9	-0.0065
3.6057	10.0000	944.5703	944.6	-0.0297
3.9564	11.0000	946.4188	946.5	-0.0812
4.3043	12.0000	948.4379	948.5	-0.0621
4.6493	13.0000	950.6260	950.7	-0.0740
4.9911	14.0000	952.9816	953.0	-0.0184
5.3295	15.0000	955.5029	955.5	0.0029
5.6644	16.0000	958.1883	958.2	-0.0117
5.9954	17.0000	961.0357	961.0	0.0357
6.3226	18.0000	964.0433	963.9	0.1433
6.6457	19.0000	967.2090	967.0	0.2090
6.9645	20.0000	970.5306	970.3	0.2306
7.2789	21.0000	974.0061	973.7	0.3061
7.5889	22.0000	977.6330	977.3	0.3330
7.8941	23.0000	981.4091	981.0	0.4091
8.1946	24.0000	985.3319	984.9	0.4319
8.4902	25.0000	989.3992	988.9	0.4992
8.7809	26.0000	993.6082	993.0	0.6082
9.0665	27.0000	997.9566	997.3	0.6566
9.3469	28.0000	1002.4417	1001.7	0.7417
9.6222	29.0000	1007.0609	1006.3	0.7609
9.8922	30.0000	1011.8117	1011.0	0.8117
10.1570	31.0000	1016.6913	1015.8	0.8913
10.4164	32.0000	1021.6972	1020.8	0.8972
10.6704	33.0000	1026.8265	1025.9	0.9265
10.9190	34.0000	1032.0767	1031.1	0.9767

TABLE VI ScS WAVES

I_0	$\Delta°$	T (sec.)	T_t (sec.)	$T - T_t$
11.1622	35.0000	1037.4450	1036.4	1.0450
11.4000	36.0000	1042.9288	1041.8	1.1288
11.6324	37.0000	1048.5253	1047.4	1.1253
11.8593	38.0000	1054.2319	1053.0	1.2319
12.0809	39.0000	1060.0458	1058.8	1.2458
12.2971	40.0000	1065.9644	1064.6	1.3644
12.5079	41.0000	1071.9851	1070.5	1.4851
12.7135	42.0000	1078.1053	1076.5	1.6053
12.9137	43.0000	1084.3222	1082.6	1.7222
13.1087	44.0000	1090.6334	1088.8	1.8334
13.2984	45.0000	1097.0363	1095.1	1.9363
13.4830	46.0000	1103.5282	1101.5	2.0282
13.6626	47.0000	1110.1068	1108.0	2.1068
13.8370	48.0000	1116.7696	1114.5	2.2696
14.0065	49.0000	1123.5140	1121.1	2.4140
14.1710	50.0000	1130.3377	1127.8	2.5377
14.3307	51.0000	1137.2382	1134.6	2.6382
14.4856	52.0000	1144.2134	1141.5	2.7134
14.6358	53.0000	1151.2608	1148.4	2.8608
14.7813	54.0000	1158.3782	1155.4	2.9782
14.9222	55.0000	1165.5633	1162.5	3.0633
15.0587	56.0000	1172.8140	1169.5	3.3140
15.1906	57.0000	1180.1280	1176.8	3.3280
15.3183	58.0000	1187.5034	1184.1	3.4034
15.4416	59.0000	1194.9380	1191.4	3.5380
15.5608	60.0000	1202.4297	1198.8	3.6297
15.6758	61.0000	1209.9766	1206.2	3.7766
15.7868	62.0000	1217.5768	1213.7	3.8768
15.8938	63.0000	1225.2282	1221.2	4.0282
15.9969	64.0000	1232.9291	1228.8	4.1291
16.0962	65.0000	1240.6776	1236.4	4.2776
16.1917	66.0000	1248.4719	1244.1	4.3719
16.2836	67.0000	1256.3102	1251.8	4.5102
16.3719	68.0000	1264.1908	1259.6	4.5908
16.4568	69.0000	1272.1120	1267.4	4.7120
16.5382	70.0000	1280.0723	1275.2	4.8723

TABLE VI ScS WAVES

I_0	Δ°	T (sec.)	T_t (sec.)	$T - T_t$
16.6162	71.0000	1288.0699	1283.1	4.9699
16.6910	72.0000	1296.1033	1291.0	5.1033
16.7625	73.0000	1304.1710	1299.0	5.1710
16.8309	74.0000	1312.2716	1307.0	5.2716
16.8963	75.0000	1320.4034	1315.0	5.4034
16.9587	76.0000	1328.5651	1323.1	5.4651
17.0182	77.0000	1336.7553	1331.1	5.6553
17.0748	78.0000	1344.9727	1339.2	5.7727
17.1286	79.0000	1353.2159	1347.4	5.8159
17.1797	80.0000	1361.4837	1355.5	5.9837
17.2282	81.0000	1369.7747	1363.7	6.0747
17.2741	82.0000	1378.0877	1371.9	6.1877
17.3175	83.0000	1386.4216	1380.1	6.3216
17.3584	84.0000	1394.7752	1388.3	6.4752
17.3969	85.0000	1403.1474	1396.5	6.6474
17.4331	86.0000	1411.5369	1404.7	6.8369
17.4670	87.0000	1419.9429	1413.0	6.9429
17.4987	88.0000	1428.3641	1421.3	7.0641
17.5282	89.0000	1436.7996	1429.5	7.2996
17.5557	90.0000	1445.2484	1437.8	7.4484
17.5810	91.0000	1453.7095	1446.1	7.6095
17.6044	92.0000	1462.1820	1454.3	7.8820
17.6259	93.0000	1470.6649	1462.6	8.0649
17.6454	94.0000	1479.1574	1470.9	8.2574
17.6631	95.0000	1487.6586	1479.2	8.4586
17.6790	96.0000	1496.1676	1487.5	8.6676
17.6932	97.0000	1504.6836	1495.8	8.8836
17.7057	98.0000	1513.2058	1504.1	9.1058
17.7165	99.0000	1521.7335	1512.4	9.3335
17.7257	100.0000	1530.2658	1520.7	9.5658
17.7333	101.0000	1538.8020		
17.7394	102.0000	1547.3414		
17.7441	103.0000	1555.8833		
17.7473	104.0000	1564.4271		
17.7491	105.0000	1572.9720		
17.7495	105.8299	1580.0640		

TABLE VII PcS (ScP) WAVES

I_0	Δ°	T (sec.)	T_t (sec.)	$T - T_t$
0.0000	0.0000	724.6931	725.0	-0.3069
0.4754	1.0000	724.7555	725.1	-0.3445
0.9504	2.0000	724.9426	725.2	-0.2574
1.4245	3.0000	725.2541	725.5	-0.2459
1.8973	4.0000	725.6899	726.0	-0.3101
2.3684	5.0000	726.2494	726.5	-0.2506
2.8374	6.0000	726.9321	727.2	-0.2679
3.3038	7.0000	727.7374	728.0	-0.2626
3.7672	8.0000	728.6645	729.0	-0.3355
4.2273	9.0000	729.7125	730.0	-0.2875
4.6836	10.0000	730.8804	731.2	-0.3196
5.1357	11.0000	732.1670	732.5	-0.3330
5.5833	12.0000	733.5712	733.9	-0.3288
6.0259	13.0000	735.0917	735.4	-0.3083
6.4633	14.0000	736.7270	737.1	-0.3730
6.8951	15.0000	738.4755	738.8	-0.3245
7.3209	16.0000	740.3358	740.6	-0.2642
7.7403	17.0000	742.3061	742.6	-0.2939
8.1532	18.0000	744.3845	744.7	-0.3155
8.5592	19.0000	746.5693	746.8	-0.2307
8.9579	20.0000	748.8585	749.1	-0.2415
9.3492	21.0000	751.2500	751.5	-0.2500
9.7327	22.0000	753.7419	753.9	-0.1581
10.1082	23.0000	756.3319	756.5	-0.1681
10.4755	24.0000	759.0178	759.1	-0.0822
10.8343	25.0000	761.7975	761.9	-0.1025
11.1845	26.0000	764.6684	764.8	-0.1316
11.5258	27.0000	767.6284	767.7	-0.0716
11.8581	28.0000	770.6750	770.8	-0.1250
12.1812	29.0000	773.8057	773.9	-0.0943
12.4950	30.0000	777.0180	777.1	-0.0820
12.7993	31.0000	780.3096	780.4	-0.0904
13.0940	32.0000	783.6778	783.7	-0.0222

TABLE VII PcS (ScP) WAVES

I_0	Δ°	T (sec.)	T_t (sec.)	T - T_t
13.3790	33.0000	787.1200	787.2	-0.0800
13.6542	34.0000	790.6337	790.7	-0.0663
13.9195	35.0000	794.2164	794.2	0.0164
14.1749	36.0000	797.8653	797.8	0.0653
14.4203	37.0000	801.5780	801.5	0.0780
14.6557	38.0000	805.3517	805.2	0.1517
14.8811	39.0000	809.1840	809.0	0.1840
15.0965	40.0000	813.0721	812.9	0.1721
15.3018	41.0000	817.0135	816.8	0.2135
15.4971	42.0000	821.0055	820.8	0.2055
15.6824	43.0000	825.0457	824.8	0.2457
15.8579	44.0000	829.1315	828.8	0.3315
16.0235	45.0000	833.2603	832.9	0.3603
16.1793	46.0000	837.4296	837.0	0.4296
16.3255	47.0000	841.6370	841.2	0.4370
16.4621	48.0000	845.8800	845.4	0.4800
16.5893	49.0000	850.1562	849.6	0.5562
16.7071	50.0000	854.4631	853.8	0.6631
16.8158	51.0000	858.7986	858.1	0.6986
16.9155	52.0000	863.1602	862.4	0.7602
17.0064	53.0000	867.5457	866.7	0.8457
17.0886	54.0000	871.9529	871.0	0.9529
17.1624	55.0000	876.3797	875.4	0.9797
17.2279	56.0000	880.8239	879.8	1.0239
17.2854	57.0000	885.2835	884.2	1.0835
17.3350	58.0000	889.7566	888.6	1.1566
17.3770	59.0000	894.2411	893.0	1.2411
17.4117	60.0000	898.7352	897.4	1.3352
17.4392	61.0000	903.2371	901.9	1.3371
17.4599	62.0000	907.7450	906.3	1.4450
17.4739	63.0000	912.2573	910.7	1.5573
17.4815	64.0000	916.7722	915.2	1.5722
17.4832	64.7469	920.1451	918.4864	1.6587

TABLE VIII PKP WAVES

I_0	Δ°	T (sec.)	T_t (sec.)	$T - T_t$
17.4832	173.6341 A	1302.5815	1299.69	2.8915
17.4829	173.0000	1299.7178	1296.90	2.8178
17.4812	172.0000	1295.2019	1292.50	2.7019
17.4778	171.0000	1290.6866	1288.10	2.5866
17.4727	170.0000	1286.1724	1283.70	2.4724
17.4657	169.0000	1281.6596	1279.30	2.3596
17.4568	168.0000	1277.1489	1274.90	2.2489
17.4458	167.0000	1272.6406	1270.50	2.1406
17.4326	166.0000	1268.1353	1266.10	2.0353
17.4171	165.0000	1263.6337	1261.70	1.9337
17.3989	164.0000	1259.1363	1257.30	1.8363
17.3780	163.0000	1254.6437	1252.90	1.7437
17.3542	162.0000	1250.1568	1248.50	1.6568
17.3271	161.0000	1245.6762	1244.10	1.5762
17.2965	160.0000	1241.2028	1239.70	1.5028
17.2621	159.0000	1236.7376	1235.30	1.4376
17.2234	158.0000	1232.2815	1230.90	1.3815
17.1802	157.0000	1227.8357	1226.60	1.2357
17.1317	156.0000	1223.4014	1222.20	1.2014
17.0774	155.0000	1218.9799	1217.90	1.0799
17.0165	154.0000	1214.5728	1213.60	0.9728
16.9481	153.0000	1210.1820	1209.30	0.8820
16.8708	152.0000	1205.8094	1205.10	0.7094
16.7831	151.0000	1201.4575	1201.00	0.4575
16.6828	150.0000	1197.1291	(1196.90)	0.2291
16.5666	149.0000	1192.8279	(1192.90)	-0.0721
16.4298	148.0000	1188.5583	(1189.00)	-0.4417
16.2644	147.0000	1184.3266	(1185.20)	-0.8734
16.0547	146.0000	1180.1416	(1181.50)	-1.3584
15.7608	145.0000	1176.0187	(1178.00)	-1.9813
14.9682	144.0000 B	1172.0000	(1174.70)	-2.7000

TABLE VIII PKP WAVES

Δ°	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
173.6341 A	3.8646	2486.4	3884.7	9.6088
173.0000	3.8647	2486.3	3884.7	9.6089
172.0000	3.8650	2486.2	3884.9	9.6091
171.0000	3.8658	2485.8	3885.2	9.6095
170.0000	3.8669	2485.3	3885.7	9.6102
169.0000	3.8683	2484.6	3886.5	9.6111
168.0000	3.8703	2483.6	3887.4	9.6123
167.0000	3.8726	2482.5	3888.5	9.6137
166.0000	3.8755	2481.1	3889.9	9.6154
165.0000	3.8788	2479.5	3891.6	9.6174
164.0000	3.8827	2477.6	3893.5	9.6197
163.0000	3.8873	2475.4	3895.7	9.6224
162.0000	3.8924	2472.9	3898.2	9.6255
161.0000	3.8983	2470.0	3901.0	9.6289
160.0000	3.9050	2466.8	3904.3	9.6328
159.0000	3.9126	2463.1	3907.9	9.6372
158.0000	3.9211	2459.0	3912.0	9.6420
157.0000	3.9307	2454.4	3916.6	9.6475
156.0000	3.9414	2449.2	3921.8	9.6536
155.0000	3.9536	2443.4	3927.6	9.6603
154.0000	3.9673	2436.9	3934.2	9.6678
153.0000	3.9829	2429.5	3941.6	9.6762
152.0000	4.0006	2421.0	3950.0	9.6856
151.0000	4.0209	2411.4	3959.6	9.6961
150.0000	4.0444	2400.4	3970.6	9.7080
149.0000	4.0719	2387.5	3983.6	9.7215
148.0000	4.1049	2372.1	3998.9	9.7372
147.0000	4.1455	2353.3	4017.7	9.7556
146.0000	4.1982	2329.2	4041.9	9.7782
145.0000	4.2744	2294.6	4076.4	9.8083
144.0000 B	4.4952	2197.6	4173.5	9.8785

TABLE VIII PKP WAVES

I_0	$\Delta°$	T (sec.)	T_t (sec.)	T - T_t
14.0843	145.0000	1175.7346		
13.6996	146.0000	1179.3412		
13.3994	147.0000	1182.8619		
13.1437	148.0000	1186.3122		
13.1408	*148.0121*	*1186.3536*		
12.9171	149.0000	1189.7010		
12.7114	150.0000	1193.0348		
12.5219	151.0000	1196.3180		
12.3454	152.0000	1199.5544		
12.1797	153.0000	1202.7470		
12.0230	154.0000	1205.8983		
11.8743	155.0000	1209.0104		
11.7326	156.0000	1212.0853		
11.5969	157.0000	1215.1245		
11.4668	158.0000	1218.1297		
11.3417	159.0000	1221.1020		
11.2211	160.0000	1224.0427		
11.1047	161.0000	1226.9529		
10.9922	162.0000	1229.8337		
10.8832	163.0000	1232.6859		
10.7775	164.0000	1235.5105		
10.6748	165.0000	1238.3082		
10.5751	166.0000	1241.0799		
10.4781	167.0000	1243.8262		
10.3836	168.0000	1246.5477		
10.2915	169.0000	1249.2453		
10.2018	170.0000	1251.9193		
10.1142	171.0000	1254.5705		
10.0286	172.0000	1257.1993		
9.9848	172.5215 C	1258.5616		

TABLE VIII PKP WAVES

$\Delta°$	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
145.0000	4.7710	2082.0	4289.0	9.9333
146.0000	4.9023	2029.2	4341.9	9.9476
147.0000	5.0101	1986.8	4384.2	9.9540
148.0000	5.1058	1949.9	4421.1	9.9558
148.0121	*5.1069*	*1949.5*	*4421.5*	*9.9558*
149.0000	5.1938	1916.6	4454.4	9.9544
150.0000	5.2764	1885.8	4485.2	9.9505
151.0000	5.3550	1857.1	4514.0	9.9445
152.0000	5.4303	1829.9	4541.2	9.9367
153.0000	5.5031	1804.0	4567.1	9.9274
154.0000	5.5737	1779.2	4591.8	9.9167
155.0000	5.6425	1755.4	4615.6	9.9048
156.0000	5.7097	1732.4	4638.6	9.8917
157.0000	5.7755	1710.3	4660.8	9.8776
158.0000	5.8402	1688.7	4682.3	9.8625
159.0000	5.9037	1667.9	4703.2	9.8465
160.0000	5.9663	1647.5	4723.5	9.8297
161.0000	6.0281	1627.7	4743.3	9.8121
162.0000	6.0890	1608.4	4762.6	9.7937
163.0000	6.1493	1589.6	4781.5	9.7746
164.0000	6.2089	1571.1	4799.9	9.7548
165.0000	6.2679	1553.1	4818.0	9.7344
166.0000	6.3263	1535.4	4835.6	9.7134
167.0000	6.3842	1518.1	4852.9	9.6918
168.0000	6.4416	1501.1	4869.9	9.6696
169.0000	6.4986	1484.5	4886.6	9.6469
170.0000	6.5552	1468.1	4902.9	9.6237
171.0000	6.6114	1452.0	4919.0	9.6000
172.0000	6.6672	1436.3	4934.8	9.5758
172.5215 C	6.6962	1428.1	4942.9	9.5630

TABLE IX PKP* WAVES

I_0	$\Delta°$	T (sec.)	T_t (sec.)	$T - T_t$
13.1408	*148.0121*	*1186.3536*	(1182.7327)	(3.6209)
13.1408	148.0000	1186.3122	(1182.7)	(3.6122)
13.1403	147.0000	1182.8946	(1179.9)	(2.9946)
13.1388	146.0000	1179.4773	(1177.1)	(2.3773)
13.1362	145.0000	1176.0606	1174.4	1.6606
13.1325	144.0000	1172.6446	1171.7	0.9446
13.1278	143.0000	1169.2297	1169.0	0.2297
13.1260	142.6783	1168.1314	1168.1314	0.0000
12.8331	142.0000	1166.3000	1166.3	0.0000
12.5423	141.0000	1163.6000	1163.6	0.0000
12.3319	140.0000	1160.9000	1160.9	0.0000
12.1955	139.0000	1158.1000	1158.1	0.0000
12.0811	138.0000	1155.3000	1155.3	0.0000
11.9581	137.0000	1152.6000	1152.6	0.0000
11.8505	136.0000	1149.9000	1149.9	0.0000
11.7546	135.0000	1147.2000	1147.2	0.0000
11.6678	134.0000	1144.5000	1144.5	0.0000
11.5882	133.0000	1141.8000	1141.8	0.0000
11.5314	132.0000	1139.0000	1139.0	0.0000
11.4779	131.0000	1136.2000	1136.2	0.0000
11.4115	130.0000	1133.5000	1133.5	0.0000
11.3487	129.0000	1130.8000	1130.8	0.0000
11.3030	128.0000	1128.0000	1128.0	0.0000
11.2451	127.0000	1125.3000	1125.3	0.0000
11.1895	126.0000	1122.6000	1122.6	0.0000
11.1609	125.0000	1119.7000	1119.7	0.0000

TABLE IX PKP* WAVES

Δ°	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
148.0121	*5.1069*	*1949.5*	*4421.5*	*9.9558*
148.0000	"	"	"	"
147.0000	"	"	"	"
146.0000	"	"	"	"
145.0000	"	"	"	"
144.0000	"	"	"	"
143.0000	"	"	"	"
142.6783	*5.1069*	*1949.5*	*4421.5*	*9.9558*
142.0000	5.2168	1908.0	4463.1	9.9536
141.0000	5.3282	1866.8	4504.2	9.9468
140.0000	5.4095	1837.3	4533.7	9.9390
139.0000	5.4610	1818.9	4552.1	9.9330
138.0000	5.5034	1803.9	4567.2	9.9273
137.0000	5.5494	1787.7	4583.3	9.9205
136.0000	5.5888	1774.0	4597.1	9.9142
135.0000	5.6229	1762.1	4608.9	9.9083
134.0000	5.6529	1751.8	4619.2	9.9028
133.0000	5.6794	1742.7	4628.3	9.8978
132.0000	5.6956	1737.2	4633.8	9.8946
131.0000	5.7100	1732.3	4638.7	9.8916
130.0000	5.7296	1725.7	4645.3	9.8876
129.0000	5.7472	1719.7	4651.3	9.8838
128.0000	5.7569	1716.5	4654.5	9.8817
127.0000	5.7715	1711.6	4659.4	9.8785
126.0000	5.7848	1707.2	4663.9	9.8755
125.0000	5.7855	1706.9	4664.1	9.8753

TABLE X $PKP^{(T)}$ WAVES

I_0	$\Delta°$	T (sec.)	T_t (sec.)	$T - T_t$
9.9848	172.5215 C	1258.5616		
9.9846	172.0000	1257.2022		
9.9837	171.0000	1254.5959		
9.9818	170.0000	1251.9899		
9.9788	169.0000	1249.3846		
9.9749	168.0000	1246.7802		
9.9699	167.0000	1244.1769		
9.9637	166.0000	1241.5751		
9.9565	165.0000	1238.9750		
9.9480	164.0000	1236.3769		
9.9383	163.0000	1233.7812		
9.9274	162.0000	1231.1881		
9.9152	161.0000	1228.5980		
9.9016	160.0000	1225.0113		
9.8866	159.0000	1223.4282		
9.8702	158.0000	1220.8492		
9.8523	157.0000	1218.2746		
9.8329	156.0000	1215.7049		
9.8118	155.0000	1213.1404		
9.7891	154.0000	1210.5815		
9.7647	153.0000	1208.0287		
9.7385	152.0000	1205.4825		
9.7105	151.0000	1202.9433		
9.6805	150.0000	1200.4116		
9.6485	149.0000	1197.8879		
9.6144	148.0000	1195.3727		

TABLE X PKP$^{(T)}$ WAVES

$\Delta°$	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
172.5215 C	6.6962	1428.1	4942.9	9.5630
172.0000	6.6963	1428.1	4942.9	9.5631
171.0000	6.6969	1428.1	4943.0	9.5636
170.0000	6.6982	1428.0	4943.1	9.5647
169.0000	6.7001	1427.8	4943.2	9.5664
168.0000	6.7027	1427.6	4943.5	9.5686
167.0000	6.7061	1427.3	4943.7	9.5715
166.0000	6.7102	1426.9	4944.1	9.5750
165.0000	6.7150	1426.5	4944.5	9.5792
164.0000	6.7207	1426.1	4945.0	9.5840
163.0000	6.7271	1425.5	4945.5	9.5896
162.0000	6.7345	1424.9	4946.1	9.5958
161.0000	6.7427	1424.2	4946.8	9.6029
160.0000	6.7519	1423.4	4947.6	9.6107
159.0000	6.7620	1422.6	4948.5	9.6193
158.0000	6.7731	1421.6	4949.4	9.6288
157.0000	6.7853	1420.6	4950.4	9.6391
156.0000	6.7986	1419.5	4951.6	9.6504
155.0000	6.8130	1418.3	4952.8	9.6626
154.0000	6.8287	1416.9	4954.1	9.6758
153.0000	6.8456	1415.5	4955.5	9.6900
152.0000	6.8638	1414.0	4957.0	9.7053
151.0000	6.8834	1412.3	4958.7	9.7217
150.0000	6.9046	1410.6	4960.5	9.7393
149.0000	6.9272	1408.7	4962.4	9.7582
148.0000	6.9515	1406.6	4964.4	9.7783

TABLE X PKP$^{(T)}$ WAVES

I_0	$\Delta°$	T (sec.)	T_t (sec.)	$T - T_t$
9.5782	147.0000	1192.8666		
9.5396	146.0000	1190.3702		
9.4987	145.0000	1187.8841		
9.4553	144.0000	1185.4089		
9.4093	143.0000	1182.9453		
9.3605	142.0000	1180.4939		
9.3088	141.0000	1178.0555		
9.2540	140.0000	1175.6309		
9.1959	139.0000	1173.2210		
9.1343	138.0000	1170.8265		
9.0690	137.0000	1168.4484		
8.9995	136.0000	1166.0878		
8.9257	135.0000	1163.7458		
8.8471	134.0000	1161.4235		
8.7632	133.0000	1159.1223		
8.6735	132.0000	1156.8435		
8.5771	131.0000	1154.5889		
8.4731	130.0000	1152.3602		
8.3603	129.0000	1150.1597		
8.2370	128.0000	1147.9897		
8.1007	127.0000	1145.8534		
7.9477	126.0000	1143.7546		
7.7718	125.0000	1141.6983		
7.5609	124.0000	1139.6919		
7.2839	123.0000	1137.7479		
7.0919	122.5088	1136.8233		

TABLE X PKP$^{(T)}$ WAVES

$\Delta°$	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
147.0000	6.9776	1404.5	4966.6	9.7998
146.0000	7.0055	1402.1	4968.9	9.8227
145.0000	7.0354	1399.6	4971.4	9.8471
144.0000	7.0674	1397.0	4974.0	9.8731
143.0000	7.1017	1394.1	4976.9	9.9008
142.0000	7.1384	1391.1	4979.9	9.9303
141.0000	7.1777	1387.9	4983.2	9.9617
140.0000	7.2198	1384.4	4986.6	9.9951
139.0000	7.2650	1380.7	4990.3	10.0307
138.0000	7.3136	1376.7	4994.3	10.0687
137.0000	7.3658	1372.4	4998.6	10.1092
136.0000	7.4222	1367.9	5003.2	10.1525
135.0000	7.4830	1362.9	5008.1	10.1989
134.0000	7.5490	1357.6	5013.4	10.2485
133.0000	7.6207	1351.8	5019.2	10.3019
132.0000	7.6989	1345.6	5025.5	10.3595
131.0000	7.7848	1338.7	5032.3	10.4217
130.0000	7.8796	1331.2	5039.8	10.4894
129.0000	7.9852	1322.9	5048.1	10.5635
128.0000	8.1039	1313.6	5057.4	10.6453
127.0000	8.2393	1303.1	5068.0	10.7365
126.0000	8.3969	1290.9	5080.1	10.8399
125.0000	8.5857	1276.5	5094.5	10.9601
124.0000	8.8238	1258.6	5112.4	11.1058
123.0000	9.1573	1233.9	5137.1	11.2996
122.5088	9.4040	1216.0	5155.0	11.4352

TABLE XI PKiKP$^{(T)}$ WAVES

I_0	$\Delta°$	T (sec.)	T_t (sec.)	T - T_t
9.9848	172.5215 C	1258.5616		
9.9847	172.0000	1257.2022		
9.9843	171.0000	1254.5958		
9.9833	170.0000	1251.9896		
9.9820	169.0000	1249.3837		
9.9801	168.0000	1246.7782		
9.9778	167.0000	1244.1732		
9.9749	166.0000	1241.5689		
9.9716	165.0000	1238.9654		
9.9678	164.0000	1236.3629		
9.9635	163.0000	1233.7613		
9.9586	162.0000	1231.1610		
9.9533	161.0000	1228.5620		
9.9474	160.0000	1225.9644		
9.9410	159.0000	1223.3684		
9.9340	158.0000	1220.7742		
9.9265	157.0000	1218.1818		
9.9184	156.0000	1215.5914		
9.9098	155.0000	1213.0032		
9.9005	154.0000	1210.4173		
9.8907	153.0000	1207.8339		
9.8804	152.0000	1205.2531		
9.8694	151.0000	1202.6750		
9.8578	150.0000	1200.0998		
9.8457	149.0000	1197.5277		
9.8329	148.0000	1194.9589		
9.8195	147.0000	1192.3934		
9.8054	146.0000	1189.8315		
9.7908	145.0000	1187.2732		
9.7755	144.0000	1184.7189		
9.7595	143.0000	1182.1686		
9.7429	142.0000	1179.6225		
9.7257	141.0000	1177.0808		
9.7078	140.0000	1174.5436		
9.6892	139.0000	1172.0111		

TABLE XI PKiKP$^{(T)}$ WAVES

I_0	$\Delta°$	T (sec.)	T_t (sec.)	$T - T_t$
9.6699	138.0000	1169.4835		
9.6500	137.0000	1166.9610		
9.6294	136.0000	1164.4438		
9.6080	135.0000	1161.9320		
9.5860	134.0000	1159.4257		
9.5633	133.0000	1156.9253		
9.5399	132.0000	1154.4308		
9.5157	131.0000	1151.9425		
9.4909	130.0000	1149.4606		
9.4653	129.0000	1146.9851		
9.4390	128.0000	1144.5164		
9.4119	127.0000	1142.0545		
9.3842	126.0000	1139.5998		
9.3556	125.0000	1137.1524		
9.3264	124.0000	1134.7124		
9.2963	123.0000	1132.2801		
9.2655	122.0000	1129.8557		
9.2340	121.0000	1127.4394		
9.2017	120.0000	1125.0313		
9.1686	119.0000	1122.6317		
9.1348	118.0000	1120.2407		
9.1001	117.0000	1117.8587		
9.0647	116.0000	1115.4857		
9.0285	115.0000	1113.1219		
8.9916	114.0000	1110.7677		
8.9538	113.0000	1108.4231		
8.9152	112.0000	1106.0885		
8.8759	111.0000	1103.7639		
8.8357	110.0000	1101.4496		
8.7948	109.0000	1099.1459		
8.7530	108.0000	1096.8528		
8.7105	107.0000	1094.5707		
8.6671	106.0000	1092.2998		
8.6229	105.0000	1090.0402		

TABLE XII_1 PKIKP $\Delta = 146°$

Points	I_o	I'_3	I_c	T_E (sec.)	$k'_3 \times 10^3$
X	6.7965	45.1771	54.5065	1177.2464	6.9586
	6.5319	48.5033	55.0000	1178.4377	7.6444
	6.3946	51.5454	56.0000	1179.0311	8.1634
	6.3301	53.9273	57.0000	1179.2968	8.5113
110	6.3023	55.5191	57.7432	1179.4063	8.7184
	6.2953	56.0460	58.0000	1179.4330	8.7827
	6.2779	58.0159	59.0000	1179.4978	9.0056
	6.2722	59.8893	60.0000	1179.5181	9.1930
Y	6.2721	60.1331	60.1331	1179.5183	9.2157
	6.2749	61.6953	61.0000	1179.5092	9.3524
1216	6.2776	62.3346	61.3612	1179.5008	9.4040
	6.2843	63.4519	62.0000	1179.4810	9.4883
	6.2990	65.1716	63.0000	1179.4405	9.6039
146bis	6.3128	66.4262	63.7403	1179.4057	9.6777
	6.3182	66.8632	64.0000	1179.3930	9.7015
	6.3412	68.5335	65.0000	1179.3426	9.7827
	6.3675	70.1877	66.0000	1179.2929	9.8489
	6.3969	71.8300	67.0000	1179.2471	9.9010
	6.4291	73.4639	68.0000	1179.2080	9.9399
	6.4639	75.0924	69.0000	1179.1783	9.9660
	6.5012	76.7181	70.0000	1179.1607	9.9800
E_o	6.5281	77.8295	70.6839	1179.1569	9.9827
	6.5409	78.3433	71.0000	1179.1578	9.9821
	6.5829	79.9700	72.0000	1179.1723	9.9728
	6.6272	81.6001	73.0000	1179.2070	9.9521
	6.6740	83.2354	74.0000	1179.2648	9.9204
	6.7231	84.8773	75.0000	1179.3488	9.8777
	6.7746	86.5275	76.0000	1179.4622	9.8241
	6.8287	88.1873	77.0000	1179.6086	9.7598
	6.8854	89.8579	78.0000	1179.7918	9.6846
Z	6.8903	90.0000	78.0847	1179.8091	9.6777

TABLE XII_1 PKIKP $\Delta = 146°$

Points	I_o	$k_c \times 10^3$	R_c (km.)	v_3' (km/s)	v_c (km/s)
X	6.7965	7.9877	1406.0	9.7841	11.2310
	6.5319	8.3604	1349.9	10.3194	11.2860
	6.3946	8.6423	1309.0	10.6856	11.3124
	6.3301	8.8314	1282.2	10.9132	11.3236
110	6.3023	8.9442	1266.5	11.0421	11.3281
	6.2953	8.9792	1261.7	11.0811	11.3291
	6.2779	9.1009	1245.1	11.2130	11.3316
	6.2722	9.2033	1231.3	11.3196	11.3323
Y	6.2721	9.2157	1229.7	11.3323	11.3323
	6.2749	9.2906	1219.7	11.4074	11.3320
1216	6.2776	9.3189	1216.0	11.4352	11.3317
	6.2843	9.3651	1209.9	11.4801	11.3311
	6.2990	9.4286	1201.7	11.5405	11.3299
146bis	6.3128	9.4692	1196.4	11.5784	11.3289
	6.3182	9.4823	1194.7	11.5904	11.3285
	6.3412	9.5270	1189.0	11.6312	11.3272
	6.3675	9.5635	1184.3	11.6640	11.3260
	6.3969	9.5923	1180.6	11.6895	11.3249
	6.4291	9.6137	1177.9	11.7083	11.3241
	6.4639	9.6281	1176.1	11.7209	11.3235
	6.5012	9.6359	1175.1	11.7276	11.3232
E_o	6.5281	9.6374	1174.9	11.7289	11.3231
	6.5409	9.6370	1175.0	11.7286	11.3231
	6.5829	9.6319	1175.6	11.7241	11.3234
	6.6272	9.6205	1177.1	11.7142	11.3238
	6.6740	9.6030	1179.3	11.6989	11.3245
	6.7231	9.5794	1182.3	11.6781	11.3254
	6.7746	9.5499	1186.0	11.6518	11.3265
	6.8287	9.5144	1190.6	11.6198	11.3276
	6.8854	9.4730	1195.9	11.5819	11.3288
Z	6.8903	9.4692	1196.4	11.5784	11.3289

TABLE XII_2 PKIKP $\Delta = 146°$ (D'-Cusp.)

Points	R_c (km.)	I_0	Δ°	T (sec.)
X	1406.0	9.6045	147.7210	1194.6726
	1349.9	8.7359	132.6880	1158.4087
	1309.0	8.1765	127.5431	1147.0093
	1282.2	7.8402	125.3703	1142.4543
110	1266.5	7.6528	124.4100	1140.5079
	1261.7	7.5964	124.1539	1139.9970
	1245.1	7.4073	123.4026	1138.5218
	1231.3	7.2555	122.9169	1137.5897
Y	1229.7	7.2375	122.8662	1137.4937
	1219.7	7.1312	122.5960	1136.9856
1216	1216.0	7.0919	122.5088	1136.8233
	1209.9	7.0286	122.3828	1136.5904
	1201.7	6.9436	122.2415	1136.3321
146bis	1196.4	6.8903	122.1695	1136.2016
	1194.7	6.8734	122.1492	1136.1651
	1189.0	6.8160	122.0901	1136.0592
	1184.3	6.7700	122.0533	1135.9938
	1180.6	6.7342	122.0313	1135.9549
	1177.9	6.7077	122.0188	1135.9329
	1176.1	6.6901	122.0122	1135.9213
	1175.1	6.6807	122.0093	1135.9161
E_o	1174.9	6.6788	122.0087	1135.9152
	1175.0	6.6792	122.0088	1135.9154
	1175.6	6.6855	122.0107	1135.9187
	1177.1	6.6994	122.0155	1135.9271
	1179.3	6.7210	122.0247	1135.9432
	1182.3	6.7502	122.0404	1135.9710
	1186.0	6.7871	122.0659	1136.0161
	1190.6	6.8321	122.1052	1136.0862
	1195.9	6.8854	122.1635	1136.1908
Z	1196.4	6.8903	122.1695	1136.2016

TABLE XII_2 PKIKP $\Delta = 146°$ (D"-Cusp.)

Points	R_C (km.)	I_0	$\Delta°$	T (sec.)
X	1406.0	8.3576	102.2903	1086.2910
	1349.9	7.9826	103.1812	1093.1951
	1309.0	7.7206	105.4338	1100.6839
	1282.2	7.5543	107.9386	1107.1646
110	1266.5	7.4585	110.0000	1111.9130
	1261.7	7.4292	110.7675	1113.5961
	1245.1	7.3294	114.2038	1120.7576
	1231.3	7.2473	119.9348	1131.9302
Y	1229.7	7.2375	122.8662	1137.4937
	1219.7	7.1312	135.7773	1161.6934
1216	1216.0	7.0919	137.9378	1165.6330
	1209.9	7.0286	140.8661	1170.8878
	1201.7	6.9436	144.1673	1176.6716
146bis	1196.4	6.8903	146.0000	1179.8091
	1194.7	6.8734	146.5539	1180.7460
	1189.0	6.8160	148.3471	1183.7416
	1184.3	6.7700	149.7069	1185.9729
	1180.6	6.7342	150.7245	1187.6188
	1177.9	6.7077	151.4554	1188.7881
	1176.1	6.6901	151.9360	1189.5509
	1175.1	6.6807	152.1890	1189.9505
E_0	1174.9	6.6788	152.2378	1190.0274
	1175.0	6.6792	152.2276	1190.0115
	1175.6	6.6855	152.0585	1189.7446
	1177.1	6.6994	151.6820	1189.1484
	1179.3	6.7210	151.0919	1188.2079
	1182.3	6.7502	150.2746	1186.8936
	1186.0	6.7871	149.2076	1185.1578
	1190.6	6.8321	147.8554	1182.9261
	1195.9	6.8854	146.1616	1180.0830
Z	1196.4	6.8903	146.0000	1179.8091

TABLE $XIII_1$ PKiKP and PKIKP WAVES

	I_0	Δ°	T (sec.)	T_t (sec.)	T - T_t
	6.4771	105	1106.4803	(1103.7)	2.7803
	6.4995	106	1108.1791	(1105.5)	2.6791
	6.5206	107	1109.8835	(1107.3)	2.5835
	6.5405	108	1111.5932	(1109.2)	2.3932
	6.5591	109	1113.3079	(1111.1)	2.2079
P	6.5765	110	1115.0274	1113.0	2.0274
	6.5925	111	1116.7512	1114.9	1.8512
K	6.6073	112	1118.4790	1116.8	1.6790
	6.6207	113	1120.2105	1118.7	1.5105
i	6.6328	114	1121.9453	1120.7	1.2453
	6.6435	115	1123.6831	1122.6	1.0831
K	6.6528	116	1125.4235	1124.5	0.9235
	6.6607	117	1127.1661	1126.4	0.7661
P	6.6672	118	1128.9106	1128.3	0.6106
	6.6723	119	1130.6566	1130.2	0.4566
	6.6759	120	1132.4038	1132.1	0.3038
	6.6781	121	1134.1517	1134.0	0.1517
	6.6788	122	1135.9000	1135.9	0.0000
D'	6.6788	122.0087	1135.9152	1135.9166	-0.0014
D"	6.6788	152.2378	1190.0274	1190.1329	-0.1055
	6.6788	152	1189.6117	1189.8	-0.1883
PK	6.6776	151	1187.8634	1188.3	-0.4366
I	6.6742	150	1186.1158	1186.8	-0.6842
KP	6.6676	149	1184.3694	1185.3	-0.9306
	6.6556	148	1182.6254	1183.7	-1.0746
	6.6322	147	1180.8858	1182.1	-1.2142
E_o	6.5281	146	1179.1569	1180.5	-1.3431
	6.3180	147	1180.8326	1182.1	-1.2674
	6.1823	148	1182.4689	1183.7	-1.2311
	6.0516	149	1184.0708	1185.3	-1.2292

TABLE XIII$_1$ PKiKP and PKIKP WAVES

	Δ°	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
	105	9.9827	1174.9	5196.1	11.7289
	106	"	"	"	"
	107	"	"	"	"
	108	"	"	"	"
	109	"	"	"	"
P	110	"	"	"	"
	111	"	"	"	"
K	112	"	"	"	"
	113	"	"	"	"
i	114	"	"	"	"
	115	"	"	"	"
K	116	"	"	"	"
	117	"	"	"	"
P	118	"	"	"	"
	119	"	"	"	"
	120	"	"	"	"
	121	"	"	"	"
	122	"	"	"	"
D'	122.0087	9.9827	1174.9	5196.1	11.7289
D"	152.2378	9.9827	1134.3	5236.8	11.3231
	152	9.9828	1134.3	5236.8	"
PK	151	9.9846	1134.1	5237.0	"
I	150	9.9896	1133.5	5237.5	"
KP	149	9.9995	1132.4	5238.7	"
	148	10.0174	1130.3	5240.7	"
	147	10.0525	1126.4	5244.6	"
E_o	146	10.2122	1108.8	5262.2	"
	147	10.5503	1073.2	5297.8	"
	148	10.7809	1050.3	5320.7	"
	149	11.0130	1028.2	5342.9	"

TABLE $XIII_1$ PKiKP and PKIKP WAVES

	I_0	Δ°	T (sec.)	T_t (sec.)	$T - T_t$
	5.9197	150	1185.6384	1186.8	-1.1616
	5.7844	151	1187.1712	1188.3	-1.1288
	5.6448	152	1188.6682	1189.8	-1.1318
	5.5003	153	1190.1280	1191.2	-1.0720
	5.3505	154	1191.5495	1192.6	-1.0505
	5.1953	155	1192.9311	1193.9	-0.9689
	5.0347	156	1194.2714	1195.3	-1.0286
	4.8686	157	1195.5691	1196.6	-1.0309
	4.6973	158	1196.8227	1197.8	-0.9773
P	4.5207	159	1198.0307	1198.9	-0.8693
	4.3391	160	1199.1920	1200.0	-0.8080
K	4.1526	161	1200.3050	1201.0	-0.6950
	3.9614	162	1201.3687	1202.0	-0.6313
I	3.7658	163	1202.3817	1203.0	-0.6183
	3.5659	164	1203.3429	1204.0	-0.6571
K	3.3621	165	1204.2513	1204.9	-0.6487
	3.1546	166	1205.1058	1205.7	-0.5942
P	2.9436	167	1205.9055	1206.5	-0.5945
	2.7293	168	1206.6494	1207.2	-0.5506
	2.5121	169	1207.3369	1207.8	-0.4631
	2.2923	170	1207.9670	1208.4	-0.4330
	2.0699	171	1208.5392	1208.9	-0.3608
	1.8454	172	1209.0527	1209.3	-0.2473
	1.6189	173	1209.5072	1209.7	-0.1928
	1.3908	174	1209.9020	1210.1	-0.1980
	1.1612	175	1210.2368	1210.4	-0.1632
	0.9304	176	1210.5111	1210.6	-0.0889
	0.6986	177	1210.7249	1210.8	-0.0751
	0.4661	178	1210.8777	1210.9	-0.0223
	0.2332	179	1210.9694	1211.0	-0.0306
F	0.0000	180	1211.0000	1211.0	0.0000

TABLE XIII$_1$ PKiKP and PKIKP WAVES

	$\Delta°$	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
	150	11.2575	1005.8	5365.2	11.3231
	151	11.5197	982.9	5388.1	"
	152	11.8037	959.3	5411.7	"
	153	12.1129	934.8	5436.2	"
	154	12.4510	909.4	5461.6	"
	155	12.8218	883.1	5487.9	"
	156	13.2298	855.9	5515.1	"
	157	13.6798	827.7	5543.3	"
	158	14.1777	798.7	5572.4	"
P	159	14.7302	768.7	5602.3	"
	160	15.3455	737.9	5633.2	"
K	161	16.0335	706.2	5664.8	"
	162	16.8059	673.8	5697.3	"
I	163	17.6776	640.5	5730.5	"
	164	18.6669	606.6	5764.4	"
K	165	19.7971	572.0	5799.1	"
	166	21.0981	536.7	5834.3	"
P	167	22.6090	500.8	5870.2	"
	168	24.3821	464.4	5906.6	"
	169	26.4886	427.5	5943.6	"
	170	29.0280	390.1	5981.0	"
	171	32.1444	352.3	6018.8	"
	172	36.0537	314.1	6057.0	"
	173	41.0955	275.5	6095.5	"
	174	47.8357	236.7	6134.3	"
	175	57.2927	197.6	6173.4	"
	176	71.5038	158.4	6212.7	"
	177	95.2227	118.9	6252.1	"
	178	142.7105	79.3	6291.7	"
	179	285.2731	39.7	6331.3	"
F	180	∞	0.0	6371.0	"

TABLE $XIII_2$ PKiKP and PKIKP WAVES

	I_0	$\Delta°$	T (sec.)	T_t (sec.)	$T - T_t$
	6.8497	105	1104.7061	(1103.7)	1.0061
	6.8752	106	1106.5023	(1105.5)	1.0023
	6.8994	107	1108.3050	(1107.3)	1.0050
	6.9224	108	1110.1139	(1109.2)	0.9139
	6.9440	109	1111.9285	(1111.1)	0.8285
P	6.9643	110	1113.7487	1113.0	0.7487
	6.9832	111	1115.5739	1114.9	0.6739
K	7.0008	112	1117.4038	1116.8	0.6038
	7.0168	113	1119.2382	1118.7	0.5382
i	7.0314	114	1121.0765	1120.7	0.3765
	7.0446	115	1122.9185	1122.6	0.3185
K	7.0561	116	1124.7636	1124.5	0.2636
	7.0661	117	1126.6116	1126.4	0.2116
P	7.0746	118	1128.4620	1128.3	0.1620
	7.0813	119	1130.3143	1130.2	0.1143
	7.0865	120	1132.1682	1132.1	0.0682
	7.0899	121	1134.0233	1134.0	0.0233
	7.0917	122	1135.8790	1135.9	-0.0210
D'	7.0919	122.5088	1136.8233	1136.8668	-0.0435
D"	7.0919	137.9378	1165.6330	1166.3818	-0.7488
	7.0909	137	1163.8926	1164.6	-0.7074
	7.0860	136	1162.0374	1162.7	-0.6626
min	7.0595	135.2090	1160.5719	1161.1970	-0.6252
	6.9967	136	1162.0255	1162.7	-0.6745
	6.9416	137	1163.8495	1164.6	-0.7505
P	6.8858	138	1165.6591	1166.5	-0.8409
K	6.8268	139	1167.4537	1168.3	-0.8463
I	6.7636	140	1169.2325	(1170.1)	-0.8675
K	6.6959	141	1170.9943	(1171.9)	-0.9057
P	6.6232	142	1172.7377	(1173.7)	-0.9623
	6.5452	143	1174.4615	(1175.4)	-0.9385
	6.4618	144	1176.1643	(1177.1)	-0.9357
	6.3726	145	1177.8445	(1178.8)	-0.9555
E	6.2776	146	1179.5008	1180.5	-0.9992

TABLE $XIII_2$ PKiKP and PKIKP WAVES

	Δ°	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
	105	9.4040	1216.0	5155.0	11.4352
	106	"	"	"	"
	107	"	"	"	"
	108	"	"	"	"
	109	"	"	"	"
P	110	"	"	"	"
	111	"	"	"	"
K	112	"	"	"	"
	113	"	"	"	"
i	114	"	"	"	"
	115	"	"	"	"
K	116	"	"	"	"
	117	"	"	"	"
P	118	"	"	"	"
	119	"	"	"	"
	120	"	"	"	"
	121	"	"	"	"
	122	"	"	"	"
D'	122.5088	9.4040	1216.0	5155.0	11.4352
D"	137.9378	9.4040	1205.0	5166.0	11.3317
	137	9.4052	1204.8	5166.2	"
	136	9.4118	1204.0	5167.0	"
min	135.2090	9.4470	1199.5	5171.5	"
	136	9.5313	1188.9	5182.1	"
	137	9.6066	1179.6	5191.4	"
P	138	9.6840	1170.1	5200.9	"
K	139	9.7674	1160.2	5210.9	"
I	140	9.8581	1149.5	5221.5	"
K	141	9.9574	1138.0	5233.0	"
P	142	10.0662	1125.7	5245.3	"
	143	10.1856	1112.5	5258.5	"
	144	10.3166	1098.4	5272.6	"
	145	10.4603	1083.3	5287.7	"
E	146	10.6179	1067.2	5303.8	"

TABLE $XIII_2$ PKiKP and PKIKP WAVES

	I_0	$\Delta°$	T (sec.)	T_t (sec.)	$T - T_t$
	6.1767	147	1181.1316	1182.1	-0.9684
	6.0696	148	1182.7352	1183.7	-0.9648
	5.9563	149	1184.3100	1185.3	-0.9900
	5.8368	150	1185.8545	1186.8	-0.9455
	5.7110	151	1187.3670	1188.3	-0.9330
	5.5789	152	1188.8458	1189.8	-0.9542
	5.4406	153	1190.2893	1191.2	-0.9107
	5.2960	154	1191.6958	1192.6	-0.9042
	5.1454	155	1193.0638	1193.6	-0.8362
	4.9887	156	1194.3917	1195.3	-0.9083
	4.8262	157	1195.6778	1196.6	-0.9222
P	4.6581	158	1196.9207	1197.8	-0.8793
	4.4844	159	1198.1189	1198.9	-0.7811
K	4.3054	160	1199.2709	1200.0	-0.7291
	4.1214	161	1200.3755	1201.0	-0.6245
I	3.9325	162	1201.4313	1202.0	-0.5687
	3.7390	163	1202.4370	1203.0	-0.5630
K	3.5412	164	1203.3915	1204.0	-0.6085
	3.3393	165	1204.2937	1204.9	-0.6063
P	3.1336	166	1205.1424	1205.7	-0.5576
	2.9244	167	1205.9369	1206.5	-0.5631
	2.7118	168	1206.6760	1207.2	-0.5240
	2.4963	169	1207.3591	1207.8	-0.4409
	2.2780	170	1207.9853	1208.4	-0.4147
	2.0572	171	1208.5539	1208.9	-0.3461
	1.8342	172	1209.0643	1209.3	-0.2357
	1.6092	173	1209.5160	1209.7	-0.1840
	1.3825	174	1209.9085	1210.1	-0.1915
	1.1543	175	1210.2413	1210.4	-0.1587
	0.9249	176	1210.5140	1210.6	-0.0860
	0.6945	177	1210.7265	1210.8	-0.0735
	0.4634	178	1210.8784	1210.9	-0.0216
	0.2318	179	1210.9696	1211.0	-0.0304
F	0.0000	180	1211.0000	1211.0	0.0000

TABLE XIII$_2$ PKiKP and PKIKP WAVES

	$\Delta°$	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
	147	10.7907	1050.1	5320.9	11.3317
	148	10.9804	1032.0	5339.0	"
	149	11.1884	1012.8	5358.2	"
	150	11.4167	992.6	5378.5	"
	151	11.6673	971.2	5399.8	"
	152	11.9427	948.8	5422.2	"
	153	12.2454	925.4	5445.6	"
	154	12.5786	900.9	5470.2	"
	155	12.9459	875.3	5495.7	"
	156	13.3513	848.7	5522.3	"
	157	13.7997	821.2	5549.9	"
P	158	14.2968	792.6	5578.4	"
	159	14.8493	763.1	5607.9	"
K	160	15.4653	732.7	5638.3	"
	161	16.1547	701.5	5669.6	"
I	162	16.9293	669.4	5701.7	"
	163	17.8039	636.5	5734.6	"
K	164	18.7970	602.8	5768.2	"
	165	19.9320	568.5	5802.5	"
P	166	21.2390	533.5	5837.5	"
	167	22.7572	497.9	5873.1	"
	168	24.5393	461.8	5909.2	"
	169	26.6567	425.1	5945.9	"
	170	29.2097	387.9	5983.1	"
	171	32.3431	350.4	6020.7	"
	172	36.2742	312.4	6058.6	"
	173	41.3445	274.1	6096.9	"
	174	48.1231	235.5	6135.6	"
	175	57.6345	196.6	6174.4	"
	176	71.9281	157.5	6213.5	"
	177	95.7853	118.3	6252.7	"
	178	143.5511	78.9	6292.1	"
	179	286.9503	39.5	6331.5	"
F	180	∞	0.0	6371.0	"

TABLE $XIII_3$ PKiKP and PKIKP WAVES

	I_0	Δ°	T (sec.)	T_t (sec.)	T - T_t
	6.9758	105	1104.0768	(1103.7)	0.3768
	7.0026	106	1105.9061	(1105.5)	0.4061
	7.0281	107	1107.7421	(1107.3)	0.4421
	7.0523	108	1109.5846	(1109.2)	0.3846
	7.0752	109	1111.4333	(1111.1)	0.3333
P	7.0967	110	1113.2877	1113.0	0.2877
	7.1169	111	1115.1476	1114.9	0.2476
K	7.1356	112	1117.0125	1116.8	0.2125
	7.1529	113	1118.8821	1118.7	0.1821
i	7.1687	114	1120.7561	1120.7	0.0561
	7.1830	115	1122.6339	1122.6	0.0339
K	7.1957	116	1124.5153	1124.5	0.0153
	7.2068	117	1126.3997	1126.4	-0.0003
P	7.2162	118	1128.2868	1128.3	-0.0132
	7.2240	119	1130.1762	1130.2	-0.0238
	7.2300	120	1132.0673	1132.1	-0.0327
	7.2343	121	1133.9599	1134.0	-0.0401
	7.2368	122	1135.8532	1135.9	-0.0468
D'	7.2375	122.8662	1137.4937	1137.5458	-0.0521
D"	7.2375	122.8662	1137.4937	1137.5458	-0.0521
	7.2375	123	1137.7470	1137.8	-0.0530
	7.2358	124	1139.6407	1139.8	-0.1593
	7.2313	125	1141.5336	1141.7	-0.1664
	7.2239	126	1143.4249	1143.7	-0.2751
	7.2136	127	1145.3140	1145.6	-0.2860
P	7.2002	128	1147.1999	1147.5	-0.3001
	7.1837	129	1149.0820	1149.4	-0.3180
K	7.1639	130	1150.9593	1151.3	-0.3407
	7.1407	131	1152.8311	1153.2	-0.3689
I	7.1139	132	1154.6963	1155.1	-0.4037
	7.0835	133	1156.5541	1157.0	-0.4459
K	7.0492	134	1158.4035	1158.9	-0.4965
	7.0108	135	1160.2434	1160.8	-0.5566
P	6.9683	136	1162.0728	1162.7	-0.6272
	6.9214	137	1163.8906	1164.6	-0.7094
	6.8700	138	1165.6955	1166.5	-0.8045
	6.8137	139	1167.4864	1168.3	-0.8136
	6.7526	140	1169.2621	(1170.1)	-0.8379
	6.6863	141	1171.0212	(1171.9)	-0.8788

TABLE $XIII_3$ PKiKP and PKIKP WAVES

	Δ°	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
	105	9.2157	1229.7	5141.4	11.3323
	106	"	"	"	"
	107	"	"	"	"
	108	"	"	"	"
	109	"	"	"	"
P	110	"	"	"	"
	111	"	"	"	"
K	112	"	"	"	"
	113	"	"	"	"
i	114	"	"	"	"
	115	"	"	"	"
K	116	"	"	"	"
	117	"	"	"	"
P	118	"	"	"	"
	119	"	"	"	"
	120	"	"	"	"
	121	"	"	"	"
	122	"	"	"	"
D'	122.8662	9.2157	1229.7	5141.4	11.3323
D"	122.8662	9.2157	1229.7	5141.4	11.3323
	123	9.2158	1229.7	5141.4	"
	124	9.2180	1229.4	5141.7	"
	125	9.2237	1228.6	5142.4	"
	126	9.2330	1227.4	5143.7	"
	127	9.2462	1225.6	5145.4	"
P	128	9.2632	1223.4	5147.7	"
	129	9.2844	1220.6	5150.5	"
K	130	9.3100	1217.2	5153.8	"
	131	9.3401	1213.3	5157.7	"
I	132	9.3750	1208.8	5162.2	"
	133	9.4151	1203.6	5167.4	"
K	134	9.4607	1197.8	5173.2	"
	135	9.5122	1191.3	5179.7	"
P	136	9.5699	1184.2	5186.9	"
	137	9.6344	1176.2	5194.8	"
	138	9.7063	1167.5	5203.5	"
	139	9.7859	1158.0	5213.0	"
	140	9.8741	1147.7	5223.4	"
	141	9.9715	1136.5	5234.6	"

TABLE XIII$_3$ PKiKP and PKIKP WAVES

	I_0	Δ°	T (sec.)	T_t (sec.)	$T - T_t$
	6.6148	142	1172.7623	(1173.7)	-0.9377
	6.5377	143	1174.4840	(1175.4)	-0.9160
	6.4550	144	1176.1849	(1177.1)	-0.9151
	6.3665	145	1177.8635	(1178.8)	-0.9365
E	6.2721	146	1179.5183	1180.5	-0.9817
	6.1716	147	1181.1477	1182.1	-0.9523
	6.0649	148	1182.7500	1183.7	-0.9500
	5.9520	149	1184.3237	1185.3	-0.9763
	5.8328	150	1185.8671	1186.8	-0.9329
	5.7073	151	1187.3785	1188.3	-0.9215
	5.5754	152	1188.8564	1189.8	-0.9436
	5.4373	153	1190.2990	1191.2	-0.9010
	5.2930	154	1191.7047	1192.6	-0.8953
	5.1425	155	1193.0720	1193.9	-0.8280
P	4.9861	156	1194.3991	1195.3	-0.9009
	4.8237	157	1195.6845	1196.6	-0.9155
K	4.6557	158	1196.9268	1197.8	-0.8732
	4.4822	159	1198.1244	1198.9	-0.7756
I	4.3034	160	1199.2759	1200.0	-0.7241
	4.1194	161	1200.3799	1201.0	-0.6201
K	3.9307	162	1201.4352	1202.0	-0.5648
	3.7374	163	1202.4405	1203.0	-0.5595
P	3.5397	164	1203.3946	1204.0	-0.6054
	3.3379	165	1204.2964	1204.9	-0.6036
	3.1323	166	1205.1448	1205.7	-0.5552
	2.9232	167	1205.9389	1206.5	-0.5611
	2.7107	168	1206.6777	1207.2	-0.5223
	2.4953	169	1207.3605	1207.8	-0.4395
	2.2771	170	1207.9864	1208.4	-0.4136
	2.0564	171	1208.5548	1208.9	-0.3452
	1.8335	172	1209.0651	1209.3	-0.2349
	1.6086	173	1209.5166	1209.7	-0.1834
	1.3819	174	1209.9089	1210.1	-0.1911
	1.1538	175	1210.2415	1210.4	-0.1585
	0.9245	176	1210.5142	1210.6	-0.0858
	0.6942	177	1210.7266	1210.8	-0.0734
	0.4632	178	1210.8784	1210.9	-0.0216
	0.2317	179	1210.9696	1211.0	-0.0304
F	0.0000	180	1211.0000	1211.0	0.0000

TABLE $XIII_3$ PKiKP and PKIKP WAVES

	$\Delta°$	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
	142	10.0789	1124.4	5246.7	11.3323
	143	10.1972	1111.3	5259.7	"
	144	10.3272	1097.3	5273.7	"
	145	10.4702	1082.3	5288.7	"
E	146	10.6272	1066.3	5304.7	"
	147	10.7996	1049.3	5321.7	"
	148	10.9888	1031.3	5339.8	"
	149	11.1965	1012.1	5358.9	"
	150	11.4245	991.9	5379.1	"
	151	11.6749	970.7	5400.4	"
	152	11.9501	948.3	5422.7	"
	153	12.2527	924.9	5446.1	"
	154	12.5858	900.4	5470.6	"
	155	12.9530	874.9	5496.2	"
P	156	13.3584	848.3	5522.7	"
	157	13.8068	820.8	5550.3	"
K	158	14.3040	792.2	5578.8	"
	159	14.8566	762.8	5608.2	"
I	160	15.4727	732.4	5638.6	"
	161	16.1622	701.2	5669.9	"
K	162	16.9370	669.1	5701.9	"
	163	17.8118	636.2	5734.8	"
P	164	18.8052	602.6	5768.4	"
	165	19.9406	568.3	5802.7	"
	166	21.2479	533.3	5837.7	"
	167	22.7667	497.8	5873.3	"
	168	24.5493	461.6	5909.4	"
	169	26.6674	424.9	5946.1	"
	170	29.2213	387.8	5983.2	"
	171	32.3559	350.2	6020.8	"
	172	36.2884	312.3	6058.7	"
	173	41.3606	274.0	6097.0	"
	174	48.1417	235.4	6135.6	"
	175	57.6567	196.5	6174.5	"
	176	71.9556	157.5	6213.5	"
	177	95.8218	118.3	6252.8	"
	178	143.6056	78.9	6292.1	"
	179	287.0591	39.5	6331.6	"
F	180	∞	0.0	6371.0	"

TABLE XIII$_4$ PKiKP and PKIKP WAVES

	I_0	Δ°	T (sec.)	T_t (sec.)	$T - T_t$
	7.3177	105	1102.2443	(1103.7)	-1.4557
	7.3484	106	1104.1631	(1105.5)	-1.3369
	7.3778	107	1106.0896	(1107.3)	-1.2104
	7.4060	108	1108.0237	(1109.2)	-1.1763
	7.4329	109	1109.9649	(1111.1)	-1.1351
D"	7.4585	110	1111.9130	1113.0	-1.0870
	7.4827	111	1113.8675	1114.9	-1.0325
	7.5056	112	1115.8282	1116.8	-0.9718
	7.5270	113	1117.7946	1118.7	-0.9054
	7.5470	114	1119.7664	1120.7	-0.9336
P	7.5654	115	1121.7432	1122.6	-0.8568
K	7.5822	116	1123.7246	1124.5	-0.7754
i	7.5974	117	1125.7102	1126.4	-0.6898
K	7.6109	118	1127.6994	1128.3	-0.6006
P	7.6226	119	1129.6920	1130.2	-0.5080
	7.6325	120	1131.6874	1132.1	-0.4126
	7.6405	121	1133.6851	1134.0	-0.3149
	7.6466	122	1135.6846	1135.9	-0.2154
	7.6507	123	1137.6855	1137.8	-0.1145
	7.6526	124	1139.6871	1139.8	-0.1129
D'	7.6528	124.4100	1140.5079	1140.5790	-0.0711
D"	7.4585	110	1111.9130	1113.0	-1.0870
	7.4582	111	1113.8643	1114.9	-1.0357
P	7.4573	112	1115.8155	1116.8	-0.9845
K	7.4557	113	1117.7663	1118.7	-0.9337
I	7.4535	114	1119.7167	1120.7	-0.9833
K	7.4504	115	1121.6664	1122.6	-0.9336
P	7.4465	116	1123.6151	1124.5	-0.8849
	7.4417	117	1125.5628	1126.4	-0.8372
	7.4360	118	1127.5091	1128.3	-0.7909

TABLE XIII$_4$ PKiKP and PKIKP WAVES

	$\Delta°$	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
	105	8.7184	1266.5	5104.5	11.0421
	106	"	"	"	"
	107	"	"	"	"
	108	"	"	"	"
	109	"	"	"	"
D"	110	"	"	"	"
	111	"	"	"	"
	112	"	"	"	"
	113	"	"	"	"
	114	"	"	"	"
P	115	"	"	"	"
K	116	"	"	"	"
i	117	"	"	"	"
K	118	"	"	"	"
P	119	"	"	"	"
	120	"	"	"	"
	121	"	"	"	"
	122	"	"	"	"
	123	"	"	"	"
	124	"	"	"	"
D'	124.4100	8.7184	1266.5	5104.5	11.0421
D"	110	8.9442	1266.5	5104.5	11.3281
	111	8.9445	1266.5	5104.5	"
P	112	8.9456	1266.3	5104.7	"
K	113	8.9475	1266.1	5105.0	"
I	114	8.9502	1265.7	5105.4	"
K	115	8.9538	1265.2	5105.9	"
P	116	8.9585	1264.5	5106.5	"
	117	8.9642	1263.7	5107.3	"
	118	8.9711	1262.7	5108.3	"

TABLE $XIII_4$ PKiKP and PKIKP WAVES

	I_0	Δ°	T (sec.)	T_t (sec.)	$T - T_t$
	7.4291	119	1129.4537	1130.2	-0.7463
	7.4211	120	1131.3964	1132.1	-0.7036
	7.4119	121	1133.3369	1134.0	-0.6631
	7.4012	122	1135.2747	1135.9	-0.6253
	7.3890	123	1137.2096	1137.8	-0.5904
	7.3753	124	1139.1412	1139.8	-0.6588
	7.3597	125	1141.0689	1141.7	-0.6311
	7.3422	126	1142.9923	1143.7	-0.7077
	7.3227	127	1144.9109	1145.6	-0.6891
P	7.3009	128	1146.8241	1147.5	-0.6759
	7.2767	129	1148.7314	1149.4	-0.6686
K	7.2499	130	1150.6320	1151.3	-0.6680
	7.2203	131	1152.5253	1153.2	-0.6747
I	7.1878	132	1154.4105	1155.1	-0.6895
	7.1521	133	1156.2869	1157.0	-0.7131
K	7.1130	134	1158.1535	1158.9	-0.7465
	7.0703	135	1160.0094	1160.8	-0.7906
P	7.0238	136	1161.8538	1162.7	-0.8462
	6.9733	137	1163.6855	1164.6	-0.9145
	6.9185	138	1165.5035	1166.5	-0.9965
	6.8593	139	1167.3067	1168.3	-0.9933
	6.7954	140	1169.0938	(1170.1)	-1.0062
	6.7266	141	1170.8637	(1171.9)	-1.0363
	6.6527	142	1172.6150	(1173.7)	-1.0850
	6.5735	143	1174.3463	(1175.4)	-1.0537
	6.4888	144	1176.0563	(1177.1)	-1.0437
	6.3985	145	1177.7435	(1178.8)	-1.0565
E	6.3023	146	1179.4063	1180.5	-1.0937
	6.2002	147	1181.0433	1182.1	-1.0567
	6.0920	148	1182.6529	1183.7	-1.0471
	5.9777	149	1184.2335	1185.3	-1.0665

TABLE $XIII_4$ PKiKP and PKIKP WAVES

	$\Delta°$	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
	119	8.9793	1261.6	5109.5	11.3281
	120	8.9890	1260.2	5110.8	"
	121	9.0001	1258.7	5112.4	"
	122	9.0130	1256.9	5114.2	"
	123	9.0278	1254.8	5116.2	"
	124	9.0446	1252.5	5118.6	"
	125	9.0636	1249.8	5121.2	"
	126	9.0850	1246.9	5124.1	"
	127	9.1091	1243.6	5127.4	"
P	128	9.1362	1239.9	5131.1	"
	129	9.1664	1235.8	5135.2	"
K	130	9.2001	1231.3	5139.7	"
	131	9.2376	1226.3	5144.7	"
I	132	9.2792	1220.8	5150.2	"
	133	9.3253	1214.8	5156.3	"
K	134	9.3763	1208.2	5162.9	"
	135	9.4326	1200.9	5170.1	"
P	136	9.4947	1193.1	5177.9	"
	137	9.5632	1184.6	5186.5	"
	138	9.6385	1175.3	5195.7	"
	139	9.7213	1165.3	5205.7	"
	140	9.8123	1154.5	5216.6	"
	141	9.9122	1142.8	5228.2	"
	142	10.0218	1130.3	5240.7	"
	143	10.1420	1116.9	5254.1	"
	144	10.2738	1102.6	5268.4	"
	145	10.4182	1087.3	5283.7	"
E	146	10.5765	1071.1	5300.0	"
	147	10.7500	1053.8	5317.3	"
	148	10.9401	1035.5	5335.6	"
	149	11.1486	1016.1	5354.9	"

TABLE $XIII_4$ PKiKP and PKIKP WAVES

	I_0	$\Delta°$	T (sec.)	T_t (sec.)	$T - T_t$
	5.8571	150	1185.7834	1186.8	-1.0166
	5.7303	151	1187.3011	1188.3	-0.9989
	5.5973	152	1188.7848	1189.8	-1.0152
	5.4581	153	1190.2330	1191.2	-0.9670
	5.3126	154	1191.6440	1192.6	-0.9560
	5.1611	155	1193.0162	1193.9	-0.8838
	5.0037	156	1194.3480	1195.3	-09520
	4.8404	157	1195.6380	1196.6	-0.9620
	4.6714	158	1196.8845	1197.8	-0.9155
P	4.4970	159	1198.0860	1198.9	-0.8140
	4.3173	160	1199.2413	1200.0	-0.7587
K	4.1326	161	1200.3489	1201.0	-0.6511
	3.9430	162	1201.4075	1202.0	-0.5925
I	3.7488	163	1202.4159	1203.0	-0.5841
	3.5504	164	1203.3729	1204.0	-0.6271
K	3.3479	165	1204.2774	1204.9	-0.6226
	3.1415	166	1205.1283	1205.7	-0.5717
P	2.9317	167	1205.9247	1206.5	-0.5753
	2.7185	168	1206.6657	1207.2	-0.5343
	2.5024	169	1207.3504	1207.8	-0.4496
	2.2835	170	1207.9782	1208.4	-0.4218
	2.0621	171	1208.5481	1208.9	-0.3519
	1.8385	172	1209.0598	1209.3	-0.2402
	1.6130	173	1209.5126	1209.7	-0.1874
	1.3857	174	1209.9059	1210.1	-0.1941
	1.1570	175	1210.2395	1210.4	-0.1605
	0.9270	176	1210.5129	1210.6	-0.0871
	0.6961	177	1210.7258	1210.8	-0.0742
	0.4645	178	1210.8781	1210.9	-0.0219
	0.2324	179	1210.9695	1211.0	-0.0305
F	0.0000	180	1211.0000	1211.0	0.0000

TABLE XIII$_4$ PKiKP and PKIKP WAVES

	$\Delta°$	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
	150	11.3772	995.7	5375.3	11.3281
	151	11.6281	974.2	5396.8	"
	152	11.9035	951.7	5419.4	"
	153	12.2063	928.1	5443.0	"
	154	12.5394	903.4	5467.6	"
	155	12.9065	877.7	5493.3	"
	156	13.3115	851.0	5520.0	"
	157	13.7595	823.3	5547.7	"
	158	14.2560	794.6	5576.4	"
P	159	14.8077	765.0	5606.0	"
	160	15.4228	734.5	5636.5	"
K	161	16.1110	703.1	5667.9	"
	162	16.8843	670.9	5700.1	"
I	163	17.7573	637.9	5733.1	"
	164	18.7485	604.2	5766.8	"
K	165	19.8813	569.8	5801.2	"
	166	21.1856	534.7	5836.3	"
P	167	22.7007	499.0	5872.0	"
	168	24.4790	462.8	5908.3	"
	169	26.5919	426.0	5945.0	"
	170	29.1393	388.8	5982.3	"
	171	32.2658	351.1	6019.9	"
	172	36.1882	313.0	6058.0	"
	173	41.2471	274.6	6096.4	"
	174	48.0104	236.0	6135.1	"
	175	57.5002	197.0	6174.0	"
	176	71.7611	157.9	6213.2	"
	177	95.5637	118.5	6252.5	"
	178	143.2196	79.1	6291.9	"
	179	286.2885	39.6	6331.5	"
F	180	∞	0.0	6371.0	"

TABLE $XIII_5$ PKiKP and PKIKP WAVES

	I_0	Δ°	T (sec.)	T_t (sec.)	T - T_t
	6.6704	105	1105.5721	(1103.7)	1.8721
	6.6943	106	1107.3214	(1105.5)	1.8214
	6.7169	107	1109.0767	(1107.3)	1.7767
	6.7383	108	1110.8378	(1109.2)	1.6378
	6.7583	109	1112.6043	(1111.1)	1.5043
P	6.7771	110	1114.3758	1113.0	1.3758
	6.7945	111	1116.1521	1114.9	1.2521
K	6.8105	112	1117.9327	1116.8	1.1327
	6.8251	113	1119.7173	1118.7	1.0173
i	6.8384	114	1121.5055	1120.7	0.8055
	6.8501	115	1123.2970	1122.6	0.6970
K	6.8604	116	1125.0914	1124.5	0.5914
	6.8693	117	1126.8882	1126.4	0.4882
P	6.8766	118	1128.6872	1128.3	0.3872
	6.8823	119	1130.4878	1130.2	0.2878
	6.8866	120	1132.2898	1132.1	0.1898
	6.8892	121	1134.0926	1134.0	0.0926
	6.8903	122	1135.8960	1135.9	-0.0040
D'	6.8903	122.1694	1136.2016	1136.2220	-0.0204
D"-E	6.8903	146	1179.8091	1180.5	-0.6909
	6.8894	145	1178.0058	(1178.8)	-0.7942
	6.8859	144	1176.2029	(1177.1)	-0.8971
	6.8780	143	1174.4015	(1175.4)	-0.9985
	6.8583	142	1172.6034	(1173.7)	-1.0966
min	6.8043	141.4359	1171.5930	(1172.6846)	-1.0916
	6.7068	142	1172.5888	(1173.7)	-1.1112
	6.6056	143	1174.3309	(1175.4)	-1.0691
	6.5102	144	1176.0478	(1177.1)	-1.0522
	6.4133	145	1177.7396	(1178.8)	-1.0604
E-bis	6.3128	146	1179.4057	1180.5	-1.0943
	6.2076	147	1181.0451	1182.1	-1.0549
	6.0972	148	1182.6563	1183.7	-1.0437

TABLE $XIII_5$ PKiKP and PKIKP WAVES

	Δ°	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
	105	9.6777	1196.4	5174.6	11.5784
	106	"	"	"	"
	107	"	"	"	"
	108	"	"	"	"
	109	"	"	"	"
P	110	"	"	"	"
	111	"	"	"	"
K	112	"	"	"	"
	113	"	"	"	"
i	114	"	"	"	"
	115	"	"	"	"
K	116	"	"	"	"
	117	"	"	"	"
P	118	"	"	"	"
	119	"	"	"	"
	120	"	"	"	"
	121	"	"	"	"
	122	"	"	"	"
D'	122.1694	9.6777	1196.4	5174.6	11.5784
D"-E	146	9.6777	1170.6	5200.4	11.3289
	145	9.6790	1170.5	5200.6	"
	144	9.6838	1169.9	5201.1	"
	143	9.6950	1168.5	5202.5	"
	142	9.7227	1165.2	5205.8	"
min	141.4359	9.7995	1156.1	5215.0	"
	142	9.9412	1139.6	5231.4	"
	143	10.0929	1122.5	5248.6	"
	144	10.2401	1106.3	5264.7	"
	145	10.3941	1089.9	5281.1	"
E-bis	146	10.5589	1072.9	5298.1	"
	147	10.7371	1055.1	5315.9	"
	148	10.9308	1036.4	5334.6	"

TABLE XIII$_5$ PKiKP and PKIKP WAVES

	I_0	Δ°	T (sec.)	T_t (sec.)	T - T_t
	5.9812	149	1184.2380	1185.3	-1.0620
	5.8595	150	1185.7887	1186.8	-1.0113
	5.7317	151	1187.3068	1188.3	-0.9932
	5.5979	152	1188.7908	1189.8	-1.0092
	5.4581	153	1190.2391	1191.2	-0.9609
	5.3123	154	1191.6500	1192.6	-0.9500
	5.1605	155	1193.0221	1193.9	-0.8779
	5.0028	156	1194.3537	1195.3	-0.9463
	4.8393	157	1195.6434	1196.6	-0.9566
	4.6702	158	1196.8896	1197.8	-0.9104
P	4.4957	159	1198.0908	1198.9	-0.8092
	4.3160	160	1199.2458	1200.0	-0.7542
K	4.1312	161	1200.3530	1201.0	-0.6470
	3.9416	162	1201.4113	1202.0	-0.5887
I	3.7475	163	1202.4193	1203.0	-0.5807
	3.5491	164	1203.3759	1204.0	-0.6241
K	3.3466	165	1204.2801	1204.9	-0.6199
	3.1403	166	1205.1307	1205.7	-0.5693
P	2.9305	167	1205.9268	1206.5	-0.5732
	2.7174	168	1206.6675	1207.2	-0.5325
	2.5014	169	1207.3519	1207.8	-0.4481
	2.2826	170	1207.9794	1208.4	-0.4206
	2.0613	171	1208.5492	1208.9	-0.3508
	1.8378	172	1209.0606	1209.3	-0.2394
	1.6123	173	1209.5132	1209.7	-0.1868
	1.3851	174	1209.9064	1210.1	-0.1936
	1.1565	175	1210.2398	1210.4	-0.1602
	0.9266	176	1210.5131	1210.6	-0.0869
	0.6958	177	1210.7259	1210.8	-0.0741
	0.4643	178	1210.8781	1210.9	-0.0219
	0.2323	179	1210.9695	1211.0	-0.0305
F	0.0000	180	1211.0000	1211.0	0.0000

TABLE $XIII_5$ PKiKP and PKIKP WAVES

	Δ°	$k(r_m) \times 10^3$	r_m (km)	z_m (km)	v_m (km/s)
	149	11.1420	1016.8	5354.3	11.3289
	150	11.3727	996.1	5374.9	"
	151	11.6253	974.5	5396.5	"
	152	11.9022	951.8	5419.2	"
	153	12.2061	928.1	5442.9	"
	154	12.5402	903.4	5467.6	"
	155	12.9081	877.7	5493.4	"
	156	13.3140	850.9	5520.1	"
	157	13.7626	823.2	5547.9	"
	158	14.2597	794.5	5576.6	"
P	159	14.8120	764.8	5606.2	"
	160	15.4276	734.3	5636.7	"
K	161	16.1163	702.9	5668.1	"
	162	16.8901	670.7	5700.3	"
I	163	17.7637	637.8	5733.3	"
	164	18.7555	604.0	5767.0	"
K	165	19.8888	569.6	5801.4	"
	166	21.1938	534.5	5836.5	"
P	167	22.7096	498.9	5872.2	"
	168	24.4887	462.6	5908.4	"
	169	26.6025	425.9	5945.2	"
	170	29.1511	388.6	5982.4	"
	171	32.2790	351.0	6020.1	"
	172	36.2030	312.9	6058.1	"
	173	41.2640	274.5	6096.5	"
	174	48.0301	235.9	6135.2	"
	175	57.5239	196.9	6174.1	"
	176	71.7908	157.8	6213.2	"
	177	95.6032	118.5	6252.5	"
	178	143.2789	79.1	6292.0	"
	179	286.4071	39.6	6331.5	"
F	180	∞	0.0	6371.0	"

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